Employment and Reallocation Effects of Higher Minimum Wages[*]

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Abstract

This paper studies the employment and reallocation effects of minimum wages in Germany in a search-and-matching model with endogenous job search effort and vacancy posting, multiple employment levels, a progressive tax-transfer system, and worker and firm heterogeneity. I find that minimum wages up to 70% of the median wage significantly increase productivity, hours worked and output without reducing employment. In frictional labor markets, however, reallocation takes time whenever the minimum wage cuts deep into the wage distribution. I show that gradually implementing a high minimum wage is necessary to avoid elevated unemployment rates during the transition.

1 Introduction

Despite much controversy, minimum wages are on the rise. Many countries and US states are passing legislation or debating proposals to substantially increase their legal wage floor.[1] This comes against the backdrop of rising wage inequality, declining union coverage and a large body of empirical research on past minimum wage reforms suggesting that observed minimum wages – ranging between 30 and 60 percent of the pre-reform median full-time wage (Kaitz index[2]) – have increased wages without significantly reducing employment.[3] A recent article by Dustmann et al. (2022) even shows that existing minimum wages improve the composition of jobs by reallocating workers towards larger and more productive firms and towards full-time jobs. However, this reduced-form research leaves open what to expect from increasing the minimum wage beyond observed levels. Policy makers currently lack a comprehensive analysis of the macroeconomic and distributional implications of higher minimum wages.

This paper takes a first step towards filling this gap by studying employment and reallocation effects of counterfactually high minimum wages in a rich search-and-matching model that is consistent with the available evidence on observed minimum wage effects. Specifically, I first estimate the model to match the distribution of wages, firm productivity and employment levels using German administrative and survey data from 2011 to 2014 – the period *before* Germany introduced its first ever federal minimum wage which affected more than ten percent of jobs. I then use this initial minimum wage reform as an independent test of the estimated model and find that the model's predictions are highly consistent with the available reduced-form evidence on the short-run employment and reallocation effects of this large policy change (Dustmann et al., 2022). Finally, I quantify the short-run and long-run effects of hypothetical reforms that raise the minimum wages beyond current levels. The main result of this analysis is that minimum wages of up to 70% of the median wage significantly increase productivity, hours worked and output without reducing employment. However, I also find that these long-run effects can take several years to materialize since high minimum wages – unlike those observed in the past – may lead to significant job destruction on impact and higher unemployment during the transition. I show that policy makers can exploit the long-run benefits of reallocation without high transitional unemployment by gradually increasing the minimum wage.

The analysis is based on a search-and-matching model of the labor market with substantial worker and firm heterogeneity, differences in employment levels (marginal, part-time, full-time), a progressive tax-and-transfer system, and endogenous search effort and vacancy posting. As firms' vacancy posting and workers' job search decisions are affected in opposite directions (Acemoglu, 2001; Flinn, 2006), the net employment effect of higher minimum wages is ambiguous.

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[1]For example, US President Biden endorsed a national minimum wage of $15 per hour during his presidential campaign but the proposed reform does not have a majority in Congress. However, several states have already decided to increase the legal wage floor to $15. Germany recently raised the federal minimum wage to €12 per hour in October 2022 which corresponds to just under 60% of the full-time median wage. The UK minimum wage is set to increase to two thirds of the full-time median wage by 2024. In 2020, the European Commission proposed a framework to improve the adequacy of minimum wages.

[2]The Kaitz index is the ratio of the minimum to the full-time median wage multiplied by 100%. Expressing the minimum wage relative to the median full-time wage facilitates comparisons across time and across countries.

[3]Among others, Cengiz et al. (2019) show this for 138 state-level minimum wage reforms in the United States. Dustmann et al. (2022) show this for the introduction of the minimum wage in Germany in 2015. See Dube (2019) for a review of this large literature.

On the one hand, firms will lower their vacancy creation as the minimum wage cuts into match profits. On the other hand, the minimum wage increases wages, earnings and thus the surplus of finding a job, which leads unemployed workers to exert more search effort.

In addition to the number of jobs, minimum wages also affect output by changing the *composition* of jobs, which features inefficiently many low-productivity and low-hours jobs due to search frictions and workers' ability to top-up low-earnings jobs with government transfers. Raising the minimum wage increases average productivity by pushing low-productivity firms out of the market (Eckstein and Wolpin, 1990; Acemoglu, 2001). In addition, raising the minimum wage increases the average employment level, i.e. average hours worked, because low-hours jobs tend to be offered by low-productivity firms and are concentrated in the bottom of the wage distribution.[4] The shift away from marginal jobs is amplified by the fact that workers' incentive to search for full-time jobs increases in the hourly wage.

I estimate the model via the method of simulated moments using German administrative linked employer-employee as well as survey data from the years 2011 to 2014, i.e. the period where the labor market was not yet affected by a federal minimum wage. I assume that workers receive a fixed share of the match output which gives rise to the well known log linear wage equation studied in the empirical literature following Abowd et al. (1999). This allows me to estimate the distributions of worker and firm productivity using a clustered two-way fixed effect procedure (Bonhomme et al., 2019). The model not only matches well the distribution of labor market states and transition probabilities for different demographic groups, but also provides a good fit to the joint distribution of wages, firm productivity and employment levels. This is important because it determines how many and what kind of jobs are affected by different minimum wage levels, which in turn determines the scope for reallocation effects.

Before analyzing counterfactually high minimum wage levels, I evaluate the introduction of Germany's first federal minimum wage in 2015 through the lens of the estimated model. This large policy shift, which raised the minimum wage from zero to €8.5 (Kaitz index of 47%) and affected more than ten percent of jobs, acts as an ideal testing ground for the model. I find that the model's short-run predictions of (i) a null-effect on total employment, (ii) a shift from marginal to part-time and full-time jobs, and (iii) an increase in average firm productivity are qualitatively and quantitatively consistent with the effects documented by recent reduced-form studies (most notably Dustmann et al., 2022).[5] The fact that the model is consistent with independent evidence on a large and observed minimum wage reform lends credibility to the main analysis of counterfactual minimum wage levels.

In the final and most important part, I analyze the short- and long-run effects of raising the minimum wage beyond observable levels relative to the baseline economy without a minimum wage or an economy with a moderate minimum wage or one with a moderate minimum wage. Focusing on steady-state comparisons, I find that the total number of jobs is essentially unaffected by minimum wages of up to 13.0€ (70%) and falls quickly thereafter. At the same time, output grows significantly as the composition of jobs improves with higher minimum wages. At

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[4]The minimum wage introduction in Germany in 2015 affected over ten percent of jobs, but only one-third of those jobs were full-time (Dustmann et al., 2022).

[5]Other studies with results on employment effects (extensive and intensive margin) include Garloff (2016), vom Berge et al. (2016), and Caliendo et al. (2017).

a minimum wage of €13.0, where the employment effect is exactly zero, total output is 3% above the baseline level. Even as the number of jobs starts to fall, output continues to increase up to a minimum wage of €14.4 (78%).

In the presence of search frictions, however, it is crucial to study transition dynamics. I first show that the favorable long-run effects of high minimum wages are the result of a potentially painful transition process. If a high minimum wage is implemented abruptly, unemployment increases discretely as firms lay off workers whose jobs have become unprofitable. It then takes time for these workers to find a new job at a more productive employer that can afford to pay the minimum wage. For example, when switching from a minimum wage of zero to a minimum wage of €13.0 (70%) – which will not change the steady state unemployment rate – the unemployment rate increases by almost 63% (4.6 percentage points) on impact and is still about 20% higher two years after the minimum wage hike. Importantly, this inter-temporal trade-off is quantitatively relevant only for high minimum wages with a Kaitz index above about 60% as the share of unprofitable jobs increases non-linearly in the minimum wage.[6] Because of this non-linearity, suddenly implementing a high minimum wage starting from a moderate minimum wage (€8.5; 47%) does not substantially alter the transition path. Instead, I show that high minimum wages need to be implemented gradually in order to avoid high transitional unemployment. Increasing the minimum wage from zero to €8.5 on impact and then gradually raising it to €13.0 over the next five years reduces the transitory increase in the unemployment from 4.6 to 0.6 percentage points.

Finally, I analyze the (long-run) distributional effects of higher minimum wages. I find that the utility gains of higher minimum wages are concentrated among male workers. Women, who tend to prefer jobs with fewer weekly hours, experience increasing disutility from work as firms offer fewer vacancies for low-hours jobs. This disutility (partly) offsets the utility gains from higher wages, earnings and consumption.[7] In addition, I find that the composition of the unemployed changes as low-skill workers find it relatively harder to find a job and, for sufficiently high minimum wages, become non-employable and stuck in long-term unemployment.

Related Literature. This paper makes several contributes to the literature on minimum wages. Despite the obvious policy relevance, this literature lacks comprehensive analyses of counterfactually high minimum wages that informs policy makers about potential reallocation effects and the turning-point where further increases in the minimum wage will lead to significant employment losses.[8] The main contribution of this paper is to offer an analysis of employment and reallocation effects of counterfactually high minimum wage levels using a rich search-and-

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[6]Hence, the model is consistent with the overarching finding in the literature that observed minimum wages (up to a Kaitz index of 60%) do not lead to significant employment effects even in the short-run (e.g. Cengiz et al., 2019; Dustmann et al., 2022). For the German minimum wage introduction in 2015, the model predicts an initial increase in the unemployment rate of less than 0.05 percentage points compared to a long-run decrease of 0.035 percentage points.

[7]Note that I interpret this disutility as a general proxy capturing not only the utility of leisure but also outside constraints such as childcare obligations.

[8]Neumark (2017) recently called for more structural research to guide minimum wage policies as extrapolating reduced-form evidence on employment effects of past (and low to moderate) minimum wage levels is a difficult undertaking. Similarly, Manning (2021) argues that the minimum wage literature should shift towards understanding at what point disemployment effects will start to kick in as we increase the minimum wage.

matching model with two-sided heterogeneity that is highly consistent with the vast reduced-form evidence of past minimum wage reforms.

Within this literature, a few recent papers study minimum wage effects in neoclassical frameworks without frictional unemployment. Ahlfeldt et al. (2021) and Bamford (2021) study the employment and welfare effects of minimum wages using a spatial equilibrium model with monopsonistic labor markets estimated using German data. Berger et al. (2021) study the (long-run) welfare and efficiency effects of minimum wages in the US using a model with oligopsonistic labor markets. While they find positive welfare gains, they find only small efficiency gains.[9]

My paper is more closely related to the strand of the literature that studies minimum wage effects in the context of search-and-matching models. Compared to early theoretical and structural research that uses rather stylized models, I adopt a more quantitative approach that leverages administrative matched employer-employee data (e.g. Burdett and Mortensen, 1998; van den Berg and Ridder, 1998; Bontemps et al., 1999; Eckstein and Wolpin, 1990; Acemoglu, 2001; Flinn, 2006). Two more recent studies also analyze minimum wage effects using quantitative search-and-matching models with two-sided heterogeneity. Blömer et al. (2020) estimate the wage posting model by Bontemps et al. (1999) in order to analyze minimum wage effects on full-time employment in Germany. Engbom and Moser (2021) estimate a wage-posting model in order to quantify the contribution of an observed increase in the minimum wage to the decline of wage inequality in Brazil.

Relative to these papers, this model has three novel ingredients that are important to understand employment and reallocation effects. First, I quantify employment effects when both vacancy posting and search effort are optimally chosen by firms and workers respectively.[10] Second, I add an intensive employment margin accounting for the fact that a very large share of minimum wage jobs is non-full-time jobs.[11] I show that both margins of reallocation can give rise to significant output effects of higher minimum wages.[12] Third, I add a progressive tax-and-transfer system, as is the case in most developed countries.[13] While providing insurance, these redistributional systems subsidize low-earnings jobs leading to disproportionately many low-hours and low-productivity jobs in the lower skill segments. This paper highlights that the reallocation effects of the minimum wage can partly offset this initial distortion induced by the tax-and-transfer system.

Besides these novel model ingredients, this paper goes beyond the existing minimum wage literature in terms of its approach. First, the particular setting in Germany allows me to cleanly bridge the gap between reduced-form and structural analyses of minimum wages. In particular,

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[9]The muted efficiency effect in Berger et al. (2021) results from reallocation *down* the productivity ladder as low productivity firms whose wages increase because of the minimum wage now attract more workers.

[10]A notable exception is Acemoglu (2001) who theoretically shows that endogenous search effort can mute disemployment effects of minimum wages but does not quantify the contribution of this channel in an estimated model.

[11]In Germany, full-time employment accounted for only one third of the jobs affected by the initial minimum wage of €8.5 and less than half of the jobs between €8.5 and €12.5 EUR in 2014.

[12]A recent paper by Doppelt (2019) shows theoretically and using reduced form evidence that higher minimum wages induce workers to work longer hours. However, the paper does not quantify the output effects of this mechanism in the context of a richer model.

[13]While countries differ in the degree of redistribution and generosity of transfers for low-earnings jobs, most countries have some sort of top-up scheme in place. The US, for example, uses in-kind transfers (e.g. food stamps) and earned income tax credits to help workers without earnings or with low earnings.

I can estimate the model on matched employer-employee data from a period where the economy was not distorted by a minimum wage (2011–2014) and then use the recent empirical evidence on a large observed minimum wage hike (2015) as an independent model test before analyzing the effects of counterfactual minimum wage levels. Second, while previous papers only compare steady states, this is the first paper to investigate the entire transition path of hypothetical minimum wage reforms revealing that short- and long-run effects may differ substantially when the minimum wage cuts sufficiently deep into the wage distribution. The key insight is that reallocation towards more productive firms takes time in the presence of search frictions.

In parallel work, Hurst et al. (2022) study the short- and long-run effects of the $15 minimum wage proposal in the US using a directed search model where homogeneous firms operate a putty-clay technology. In contrast to my results, they find substantial short-run and dramatic long-run disemployment effects of moderate and high minimum wages because firms substitute high-skill labor and capital for low-skill labor. While their analysis focuses on sluggish adjustment of inputs within homogeneous firms, this paper studies reallocation across firms with heterogeneous productivity (and jobs with different employment levels). These papers are therefore highly complementary.[14]

This paper is also related to the vast empirical literature evaluating past reforms (e.g. Cengiz et al., 2019; Dustmann et al., 2022).[15] My model rationalizes the "elusive employment effect" (Manning, 2021), i.e. the finding that past minimum wages (up to a Kaitz index of 60%) have not had a significant impact on total employment (positive or negative). In particular, my model generates a very small effect on total employment for *all* previously observed minimum wage levels as the net effect of workers' search and firms' vacancy responses.[16] In addition, this paper quantitatively rationalizes the reallocation patterns away from marginal jobs and towards more productive firms observed following the German minimum wage introduction in 2015.

Finally, by including endogenous search effort, my paper is related to the literature on employment effects of other labor market policies that target the surplus of employment. The large literature on unemployment benefits has worked to understand how benefits or benefit duration affect employment by influencing workers' incentives to exert search effort and find a job (e.g. Baily, 1978; Ljungqvist and Sargent, 1998; Chetty, 2008; Krause and Uhlig, 2012; Krebs and Scheffel, 2013; Schmieder et al., 2016; Landais et al., 2018; Hagedorn et al., 2019; Price, 2018). While these papers study how the surplus of employment evolves when unemployment benefits change, I analyze how minimum wages affect employment because the value of employment is affected by the minimum wage.

Outline. The remainder of the paper is structured as follows. Section 2 presents the equilibrium search-matching model. Section 3 describes the estimation procedure and evaluates how the model fits the pre-reform data. Section 4 analyzes the introduction of the German minimum wage and compares the model's predictions to the findings of the empirical literature. Section 5

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[14]As for the afore-mentioned search-and-matching models, Hurst et al. (2022) also do not include an intensive employment margin or a progressive tax- and transfer system.

[15]Empirical studies of the German minimum wage introduction in 2015 include Garloff (2016); vom Berge et al. (2016); Bossler and Gerner (2016); Caliendo et al. (2017); Ahlfeldt et al. (2018); Dustmann et al. (2022).

[16]See Cengiz et al. (2019) for evidence on this null-effect on total employment independent of the minimum wage level (below a Kaitz index of 60%). See Dube (2019) for a review of the empirical minimum wage literature.

analyzes counterfactually high minimum wages. Finally, section 6 concludes and discusses areas for future research.

2 Model

I study an economy where a unit mass of workers meet a mass m_f of firms in a labor market with search frictions. Time is discrete and both workers and firms are infinitely-lived. Workers differ by human capital and demographics, and firms differ by productivity. Both worker and firm heterogeneity is exogenous and time-invariant.

2.1 Workers

Workers differ by gender and family status. In particular, I distinguish between the following five demographic groups indexed by j: married men, single men, single women with and without kids, and married women.[17] Let P_j denote the population share of group j. A worker's demographic type determines her preferences over employment levels as well as her tax-and transfer schedule.[18]

Workers further differ by their time-invariant human capital (skill) h. The gender-specific distribution function of human capital is $\Phi^{g(j)}$ where g is the gender of group j. I assume that the labor market is segmented with respect to workers' skill levels such that there is a continuum of independent labor markets – one for each level of h (van den Berg and Ridder, 1998; Engbom and Moser, 2018).

A type-j worker with human capital h can be employed, $s = e$, short-term unemployed, $s = su$ or long-term unemployed, $s = lu$. There are three *employment levels* (hours worked) which I label full-time ($x = ft$), part-time ($x = pt$) and marginal employment ($x = mj$).[19] In addition, jobs differ with respect to the employer's productivity p which will be described below. While short-term unemployed workers receive unemployment insurance proportional to their previous earnings, all long-term unemployed workers receive the same unemployment benefits, i.e. a subsistence minimum. In sum, for each skill level h there is a continuum of idiosyncratic states for employed and short-term unemployed workers and a single state for long-term unemployment. The state space of a type-j worker with human capital h is

$$\mathcal{S} = \Big\{ \{(s,x,p) \mid s \in \{e, su\}, x \in \{ft, pt, mj\}, p \geq 1\}, lu \Big\}$$

In the following, I denote by σ one point in the state-space of a worker ($\mathcal{S}$) and F the distribution of endogenous states (given j and h).

When a worker with human capital h works a type-x job at a firm with productivity p, the match output is $f(h, x, p) = e_x a_x h p$ for $x \in \{ft, pt, mj\}$. The parameters e_x denote the hours worked in full-time, part-time and marginal jobs respectively, and hours worked in full-time employment are normalized to one. The parameters $a_x > 0$ allow for constant productivity

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[17]As men with and without children are similar with respect to all targeted moments, I only distinguish between single and married men. The same holds for married women. Table A.1 in the appendix shows the population shares of each demographic type.

[18]Whenever possible, I will drop the subscript j for worker types to improve readability.

[19]Marginal employment is referred to as "mini-jobs" in Germany. The monthly income of a mini-job is €450 or less and not subject to personal income taxation.

differences between full-time, part-time and marginal jobs. Workers earn a fixed and exogenous share $r \in (0,1)$ of the match output such that a worker's log wage is linear in log worker ability and log firm productivity. Hence, the model yields the simple but successful two-way fixed effect equation that has been studied by a large literature in labor economics following the seminal contribution by Abowd and Card (1989) (henceforth AKM).[20] In the presence of a minimum wage $\bar{w}$, the hourly wage is

$$w(h,x,p) = \max\{rf(h,x,p), \bar{w}\} \tag{1}$$

Gross earnings and net earnings are given by

$$\tilde{y}(h,x,p) = e_x w(h,x,p)$$
$$y^j(h,x,p) = \tilde{y}(h,x,p) - T^j(\tilde{y}(h,x,p)) \tag{2}$$

where $T^j(\tilde{y})$ is a tax function that depends on the worker's demographics. Throughout, I refer to taxes as the sum of income taxes and social insurance contributions. One particularity of marginal jobs is that workers can earn at most €450.[21]

Short-term unemployed workers receive a share b of their previous net earnings up to a maximum amount of B_{max} (unemployment insurance). Long-term unemployed workers receive subsistence benefits B_{min} independent of their skill level or previous earnings. Short-term unemployment insurance is capped from below by B_{min}. Employed workers are also eligible for unemployment benefits to top up their net earnings or unemployment insurance. In doing so, a share τ_{top} of net earnings will be deducted from B_{min}. Finally, married workers receive non-labor income y^j_{free} which is always deducted from B_{min}.[22] Hence, subsistence benefits for type-j workers may not exceed $B^j_{min} = \max\{B_{min} - y^j_{free}, 0\}$.

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[20]For Germany, Card et al. (2013) show that the AKM wage equation provides a good fit to the German wage structure find no evidence against the model's underlying assumption of exogenous mobility. In addition, recent research shows that – even for previously unemployed workers – wages of Austrian workers are insulated from the value of non-employment (Jäger et al., 2020). For Germany, Price (2018) also finds small wage effects of the cut in unemployment benefits (Hartz IV reform in 2005). In addition, (Di Addario et al., 2020) show using Italian data that the productivity of a worker's previous firm has almost no effect on the wage earned at the poaching firm. While this casts some doubt on the suitability of the Nash bargaining protocoll, the fixed piece-rate assumed here is in line with these findings. In addition, I show in Section 4 that the estimated model matches not only the pre-reform wage distribution but also the absence of significant short-run disemployment effects following the German minimum wage introduction (which is not targeted in the estimation). Wage-posting, the other standard assumption in the literature, would render the computation of equilibrium and estimation infeasible due to the presence of endogenous search effort, and multiple employment levels and demographic types. In addition, (e.g. Engbom and Moser, 2021) show that wage posting implies substantial wage spillovers after minimum wage hikes. While there is evidence for large spillovers in Brazil, Cengiz et al. (2019) and Dustmann et al. (2022) find only modest spillovers in the US and Germany.

[21]In the model, I also impose this restriction by restricting high skill workers' hours worked on a marginal job such that they earn at most €450. Dropping this restriction does not change my results significantly.

[22]The type-specific and exogenous non-labor income y^j_{free} represents a share of the partner's income for married workers. Singles do not receive such non-labor income.

As there is no savings device, consumption c equals net income.[23] A type-j worker with skill h faces the following consumption schedule

$$c^j(h, \sigma) = \begin{cases} y^j(h, x, p) + \max\left\{B^j_{min} - \tau_{top}y^j(h, x, p), 0\right\} + y^j_{free} & \text{if } s = e \\ by^j(h, x, p) + \max\{B^j_{min} - by^j(h, x, p), 0\} + y^j_{free} & \text{if } s = su \\ B^j_{min} + y^j_{free} & \text{if } s = lu \end{cases} \quad (3)$$

where $\sigma \in \mathcal{S}$ denotes one state in the worker's state space.

Workers exert costly search effort ℓ to find (better) jobs in their skill segment of the labor market. A worker in employment state s meets a vacancy with probability

$$\lambda_\sigma(\ell|h) = \phi^\sigma \ell \Lambda(\theta_h) \quad (4)$$

where labor market tightness θ_h is taken as given and ϕ^σ is a search efficiency parameter. I will assume that search efficiency differs by employment level and between short- and long-term unemployed ($\phi^{su}, \phi^{lu}, \phi^{ex}$). Importantly, not every meeting has to result in a match since search cannot be directed towards certain employment levels or high-productivity firms, and workers may decline lower-valued offers.

The mass of search-weighted workers of type-j is denoted by $S^j(h)$ and the mass of all search-weighted workers in skill segment h is

$$S(h) = \sum_j P_j \underbrace{\int_\sigma \phi^\sigma \ell(\sigma|j, h) \mathrm{d}F(\sigma|j, h)}_{S^j(h)} \quad (5)$$

where $\ell(\cdot|j, h)$ and $F(\cdot|j, h)$ represent the optimal search effort and stationary distribution functions for type-j workers in skill segment h.

Workers' utility depends on consumption, the employment level and job search:

$$u^j(\ell|h, \sigma) = \tilde{u}(c^j(h, \sigma)) - d(\ell) + \nu^j(x(\sigma)) \quad (6)$$

Here, $\tilde{u}(c)$ is a concave flow utility function of consumption, $d(\ell)$ is a convex search cost function and $\nu^j(x(\sigma))$ captures the (dis-)utility of different employment levels relative to nonemployment. The latter may depend on workers' demographics j. Single women with kids may, for example, have a strong preference for part-time or marginal jobs.[24] Heterogeneity in $\nu^j(x)$ will allow the model to match the joint distribution of employment levels and demographics.

2.2 Firms

There is a mass m_f of risk-neutral firms with heterogeneous productivity $p \sim \Gamma$. Firms employ workers of all skill levels h at all employment levels x. I assume that firms operate a linear production technology such that total output of a firm with productivity p is the sum of the

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[23]This assumption is justified by the focus on the bottom of the wage distribution.

[24]I emphasize that these "preference" parameters not only capture the tastes for leisure, but also exogenous constraints such as childcare obligations. As I do not explicitly model policies affecting child care constraints, using such a proxy is justified even though the parameter is not policy-invariant outside the model.

match outputs

$$\sum_x \int_{\underline{h}}^{\bar{h}} f(h,x,p)L(h,x,p)\mathrm{d}h$$

where $L(h,x,p)$ is the firm's mass of employees with skill h and demographics j working a type-x job. This implies that there are no complementarities between low- and high-skill workers.[25]

Firms attract workers for type-x jobs in skill segment h by posting vacancies $v(h,x)$ at a convex cost $\kappa_x(h,v)$. As hiring a worker does not affect future recruitment, firms will not reject workers of a particular demographic type even if different workers are more or likely to switch employers than others. Denote by $N(h,x)$ the mass of type-x vacancies in skill segment h and the total number of vacancies as $N(h) = \sum_x N(h,x)$. In addition, let $\Psi(h)$ denote the distribution of employment levels and productivities among all vacancies in skill segment h. Firms' vacancy posting response to a binding minimum wage can affect both the $N(h)$ and $\Psi(h)$. The former impacts labor market tightness, job finding probabilities and the total number of jobs. The latter will determine the composition of jobs and thus the average productivity and employment level.

2.3 Labor Market

Recall that labor markets are segmented by worker skill h and workers cannot direct search towards a certain employment level or towards high-productivity firms. Hence, the total mass of search and vacancies in a skill segment are matched by the matching function

$$M(h) = N(h)^\xi S(h)^{1-\xi} \tag{7}$$

where ξ is the elasticity of matches with respect to the mass of posted vacancies. Labor market tightness is defined as

$$\theta(h) = \frac{N(h)}{S(h)} \tag{8}$$

and the aggregate contact rates for a unit of search and a vacancy are $\Lambda(\theta) = \theta^\xi$ and $\Pi(\theta) = \theta^{\xi-1}$, respectively.

Employment relationships are terminated for three mutually exclusive reasons. First, workers may voluntarily change firms and/or employment levels as a result of on-the-job search. In equilibrium, firms with low productivity will be more likely to experience this event.

Second, workers may be hit by a so-called Godfather shock which forces them to switch to a different job that is randomly drawn from the distribution of vacancies. This is important to account for the substantial share of job-to-job transitions that are accompanied by a wage cut and cannot be explained by on-the-job search (Jolivet et al., 2006). The Godfather shock

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[25]This assumption is rather standard in papers studying frictional labor markets (e.g. Bagger et al., 2014; Bagger and Lentz, 2018). The assumption is also supported by the findings of Cengiz et al. (2019) who demonstrate that the minimum wage elasticity for higher-skilled employment should be very small with a neoclassical production function and plausible parameter values for the elasticity of substitution between low- and high-skill workers.

arrives with probability $\pi_{e|e_x}(h) = \psi_x \Lambda(\theta)$ and captures involuntary and unintended job-to-job transitions unrelated to workers' search effort. These may be the result of firms' outplacement programs, workers' search effort after an advance-notice layoff or family-related events that force workers to move and look for a new job immediately.

Third, matches can be destroyed such that the worker transitions into short-term unemployment. This happens with probability $\pi_{su|e_x}$ and if a minimum wage hike makes the match unprofitable for the firm.

2.4 Worker Problem

Workers choose search effort ℓ and reject or accept job offers in order to maximize discounted lifetime utility. Labor market tightness and the distribution of vacancies are taken as given.

The value of long-term unemployment for a type-j worker with human capital h solves the following Bellman equation:

$$V_{lu}^j(h) = \max_\ell \left\{ u^j(\ell, h, lu) + \beta \lambda_{lu}(\ell|h) \mathbb{E}_{(x,p)} \left[\max \left\{ V_e^j(h, x, p), V_{lu}^j(h) \right\} \middle| h \right] \right. \\ \left. + \beta \big(1 - \lambda_{lu}(\ell|h)\big) V_{lu}^j(h) \right\} \tag{9}$$

Search effort ℓ is associated with lower flow utility but a higher probability of meeting a firm. Upon meeting a firm offering a (x, p) job, the worker accepts the job if and only if the value of the employment relationship, $V_e^j(h, x, p)$, exceeds the value of remaining long-term unemployed. The max-operator in the continuation value captures this acceptance decision. The expectation is taken with respect to the distribution of vacancies in the worker's skill segment. With probability $1 - \lambda_{lu}(\ell|h)$, the worker does not meet a firm and remains long-term unemployed.

The value of short-term unemployment when the previous job was of type x at a type-p firm is

$$V_{su}^j(h, x, p) = \max_\ell \left\{ u^j(\ell|h, (su, x, p)) + \beta \pi_{lu|su} V_{lu}^j(h) \right. \\ \left. + \beta \lambda_{su}(\ell|h) \mathbb{E}_{(x',p')} \left[\max \left\{ V_e^j(h, x', p'), V_{su}^j(h, x, p) \right\} \middle| h \right] \right. \\ \left. + \beta \big(1 - \pi_{lu|su} - \lambda_{su}(\ell|h)\big) V_{su}^j(h, x, p) \right\} \tag{10}$$

The only difference to long-term unemployment is that the worker transitions from short- to long-term unemployment with exogenous probability $\pi_{lu|su}$.

The value of a worker employed at a type-p firm on a type-x job is

$$V_e^j(h, x, p) = \max_\ell \left\{ u^j(\ell|h, (e, x, p)) + \beta \pi_{su|e_x} V_{su}^j(h, x, p) \right. \\ \left. + \beta \lambda_{e_x}(\ell|h) \mathbb{E}_{(x',p')} \left[\max \left\{ V_e^j(h, x', p'), V_e^j(h, x, p) \right\} \middle| h \right] \right. \\ \left. + \beta \pi_{e|e_x}(h) \mathbb{E}_{(x',p')} \left[V_e^j(h, x', p') \middle| h \right] \right. \\ \left. + \beta \big(1 - \pi_{su|e_x} - \lambda_e(\ell|h) - \pi_{e|e_x}(h)\big) V_e^j(h, x, p) \right\} \tag{11}$$

Employed workers become short-term unemployed with probability $\pi_{su|e_x}$, receive a job offer that they can decline through on-the-job search with probability $\lambda_{e_x}(\ell|h)$ and are involuntarily reallocated to a different job with probability $\pi_{e|e_x}(h)$.

All workers may have an incentive to search for a (better) job. The first order condition determining optimal search effort is given by

$$\frac{\mathrm{d}d^j(\ell)}{\mathrm{d}\ell} = \beta \frac{\partial \lambda_\sigma(\ell|h)}{\partial \ell} \left(\underbrace{\mathbb{E}_{(x,p)} \left[\max\left\{ V_e^j(h,x,p), V^j(h,\sigma) \right\} \middle| h \right] - V^j(h,\sigma)}_{\text{expected surplus of meeting a firm}} \right) \tag{12}$$

For a worker in state σ, the job finding probability is the result of optimal search effort $\ell(\sigma)$ as well as the worker's acceptance decision

$$\pi^j(\ell|h,\sigma) = \lambda_\sigma(\ell|h)\mathbb{E}_{(x,p)}\left[\mathbb{1}\{V_e^j(h,x,p) > V^j(h,\sigma)\} \middle| h \right] \tag{13}$$

2.5 Firm Problem

Firms maximize expected discounted profits taking as given labor market tightness, the distribution of vacancies and the distribution of workers' search effort. As total production is additive in h and x, the firm faces a sequence of independent optimization problems – one for each (h,x)-segment. Each period, firms post vacancies, which may result in an employment relationship starting in the subsequent period. Unfilled vacancies are not carried over to the next period but have to be re-posted. Additive production combined with the fact that the cost of posting vacancies is independent of the current workforce further implies that the firm's optimal amount of vacancies is independent of the current workforce. For the same reasons, firms will not reject workers of a particular demographic type.

A type-x employment relationship with a type-j employee may be dissolved either due to exogenous job destruction, a Godfather shock or on-the-job search with probability:

$$\delta^j(h,x,p) = \pi_{su|e_x} + \pi_{e|e_x}(h) + \pi^j(\ell(\sigma)|h) \tag{14}$$

The probability of filling a vacancy is equal to the aggregate contact rate times the probability that the contacted worker accepts the offer:

$$\eta(h,x,p) = \Pi(\theta_h)\frac{S(h,x,p)}{S(h)} \tag{15}$$

Here, $S(h)$ is the total search-weighted mass of workers in skill segment h and $S(h,x,p)$ is the mass of search-weighted workers in segment h willing to accept a type-x job at a firm with productivity p:

$$S(h,x,p) = \sum_j S^j(h,x,p) \tag{16}$$

$$S^j(h,x,p) = P_j \int_\sigma \phi_\sigma \ell(\sigma|j,h)\mathbb{1}\{V_e^j(h,x,p) > V^j(h,\sigma)\}\mathrm{d}F(\sigma|j,h) \tag{17}$$

Let $(1-r^+)$ be the firm's profit share of the match output. If the minimum wage is binding for a (h, x, p)-job, $(1-r^+)$ is lower than the baseline profit share, $(1-r)$. Given r^+, the value $W^j(h, x, p)$ of a type-x employment relationship with a worker of type j in segment (h, x) for a firm with productivity p is given by

$$
\begin{aligned}
W^j(h, x, p) &= \underbrace{(1 - r^+) f(h, x, p)}_{\text{flow profit}} + \beta^f \big(1 - \delta^j(h, x, p)\big) W^j(h, x, p) \\
&= \frac{(1 - r^+) f(h, x, p)}{1 - \beta^f \big(1 - \delta^j(h, x, p)\big)}
\end{aligned}
\tag{18}
$$

where β^f is the firms' discount factor. When posting a vacancy, the firm has to take the expectation over worker types as they differ in their on-the-job search effort, which affects the separation probability and expected value of a match. The ex-ante expected value of filling a vacancy is thus

$$
\begin{aligned}
\mathbb{E}\big[W(h, x, p)\big] &= \sum_j \frac{S^j(h, x, p)}{S(h, x, p)} W^j(h, x, p) \\
&= (1 - r^+) f(h, x, p) \underbrace{\sum_j \frac{S^j(h, x, p)}{S(h, x, p)} \frac{1}{1 - \beta^f \big(1 - \delta^j(h, x, p)\big)}}_{\text{discounted expected match duration}}
\end{aligned}
\tag{19}
$$

Knowing the expected value of an employment relationship, the optimal number of vacancies has to satisfy

$$
\kappa'(v, h, x) = \beta^f \eta(h, x, p) \mathbb{E}\big[W(h, x, p)\big]
\tag{20}
$$

Optimal vacancy posting then requires firms to post vacancies until the marginal cost of posting another vacancy is equal to the discounted expected value of an employment relationship weighted by the probability of filling the vacancy.

2.6 Equilibrium

A stationary equilibrium consists of value functions, $V_{lu}^j(h)$, $V_{su}^j(h, x, p)$, $V_e^j(h, x, p)$, search effort policy functions, $\ell^j(h, \sigma)$, vacancy posting policy functions, $v(h, x, p)$, labor market tightness, $\theta(h)$, a distribution of vacancies, $\Psi(h, x, p)$, and a distribution of workers across states, $F^j(h, \sigma)$, that satisfy the following conditions. First, given labor market tightness and the distribution of vacancies, the value and search effort policy functions solve the workers' problem (equations 9, 10, 11, and 12). Second, given labor market tightness, workers' search policies and value functions, and the distribution of workers across states, firms' vacancy posting policy functions solve the firms' optimality conditions (equation 20). Third, the distribution of workers across states is stationary. That is, given the economy starts at this distribution and given the policy functions and labor market tightness, the distribution of workers across states will not change.

3   Estimation

In this section, I first describe the pre-set parameters and parameterize workers' flow utility and skill distributions, firms' productivity distribution and vacancy posting cost function and the tax schedule (section 3.1). Second, I discuss which moments I target in the method of simulated moments in order to identify the jointly estimated parameters (section 3.2). Third, I evaluate the estimation results and model fit (section 3.4).

3.1   Parameterization and Pre-Set Parameters

One period in the model corresponds to one quarter. I set the quarterly discount factor of both workers and firms equal to $\beta = 0.98$ and choose the minimum wage of €8.5 as the numéraire.

The employment level for full-time employment, e_{ft} is normalized to one and e_{pt} and e_{mj} are set to match the ratio of average weekly hours of part-time and marginal workers relative to full-time employed workers reported by Dustmann et al. (2022) who have access to hours worked in the German social security data. This yields $e_{pt} = 0.615$ and $e_{mj} = 0.223$.

I set $r_{ft} = r_{pt} = 0.62$ which approximately matches the aggregate labor share in Germany between 2011 and 2014. The labor share for marginal jobs r_{mj} is estimated and allowed to be lower in order to match the joint distribution of wages and employment levels. As marginal jobs constitute a tiny share of the aggregate wage bill, this does not affect the labor share significantly. The vacancy-elasticity of the matching function, ξ, is set to 0.3 following the literature review by Petrongolo and Pissarides (2001). In Appendix A, I show that the main insights on employment and reallocation effects of counterfactually high minimum wages are robust to assuming alternative values of the piece rate and the vacancy-elasticity of matches.

The German transfer system distinguishes between short- and long-term unemployment. During the first year of unemployment, workers are paid a fixed fraction $b = 0.6$ of their previous earnings (ALG I), but not less than the subsistence minimum B_{min}. With a constant net replacement rate for short-term unemployed workers, benefits differ by previous earnings. Long-term unemployed workers receive the subsistence minimum B_{min} independent of their previous earnings (ALG II). I set the policy parameter B_{min} to €800 which corresponds to about 55% of of full-time monthly earnings at the minimum wage of €8.5. For employed workers, 80% of their net earnings is deducted from the amount of subsistence benefits they are eligible to receive on top of their earnings ($\tau_{top} = 0.8$). Hence, all workers with monthly net earnings of at least €1,000 are not eligible for top-up transfers. Workers with net earnings below this threshold are eligible for subsistence transfers if they do not receive non-labor income y_{free}^{j} from their spouse. Using SOEP data that allow me to link spouses, I calculate average net earnings of the spouses of the married men and women in my sample. I then assign half of that amount to the spouse as non-labor income. On average, married women have roughly €894 and married men €409 in non-labor income from their spouses' net earnings. Non-labor income is deducted from subsistence benefit eligibility. With $B_{min} = 800$, this implies that married women are not eligible for subsistence benefits and married men receive at most half of total subsistence benefits. Singles are assumed to have no non-labor income and are hence eligible for the full amount of subsistence benefits.

I assume that workers pay a constant marginal tax rate τ^j on earnings above an exemption level D^j.

$$y_{net} = \min\{y_{gross}, D^j\} + (1 - \tau^j)\max\{0, y_{gross} - D^j\} \tag{21}$$

and estimate the parameters on SOEP data for gross and net earnings for the years 2013 and 2014 separately for different the socioeconomic worker types. Figure A.1 in Appendix A shows that the estimated average tax function provides a good fit to the binned data.

I assume that firm productivity $p \geq 1$ is drawn from a Log Gamma distribution with shape and scale parameters α and θ. Productivity differences across job types are governed by $a_{pt}, a_{mj} \in (0, 1]$ with a_{ft} normalized to one. Human capital is drawn from a gender-specific left-truncated Log Normal distribution defined by μ_h^g and σ_h^g. The truncation bound h_{min} is chosen such that the lowest possible wage – resulting from a match between the least productive firm ($p_{min} = 1$) and lowest skilled worker generates a wage of €4 i.e. $rh_{min}p_{min}a_{mj} = 4$. Data from the SOEP as well as the German Survey of Earnings Structure show that there are virtually no jobs with an hourly wage below €4 (Minimum Wage Commission, 2018).

Workers' utility depends on consumption, job search and the employment level in the following way:

$$u^j(\ell|h, \sigma) = \frac{c^j(h, \sigma)^{1-\gamma_c}}{1 - \gamma_c} - \ell^\zeta + h^\epsilon \sum_x \gamma_x^j \mathbb{1}\{x(\sigma) = x\} \tag{22}$$

where $\zeta > 1$ and γ_x^j are constants that capture the (dis-)like for the different employment levels (relative to nonemployment) for type-j workers. The state-specific constants will allow the model to match the distribution over employment levels for each demographic group. The state-constants are scaled by h^ϵ where $\epsilon > 0$ implies that the absolute importance of the state-(dis-)utilities grows with human capital. The parameter ϵ may help to match the joint distribution of wages and employment levels.[26]

Finally, I assume that the cost of posting v vacancies for type-x jobs in skill segment h is given by

$$\kappa(v, h, x) = \underbrace{e_x \kappa_1}_{\equiv \kappa_{1x}} v^{\kappa_2^x} f(h)^{1-\kappa_2^x} \tag{23}$$

where f is the density of workers' human capital and e_x is the employment level.[27] The convexity of the cost function may depend on the job type. I scale the cost of posting vacancies by the density of human capital due to the assumption of segmented labor markets. This implies that optimal vacancy creation satisfies

$$v(h, p, x) = \left(\frac{(1 - r^+)f(x, h, p)A(h, p, x)}{\kappa_{1x}\kappa_2^x}\right)^{\frac{1}{\kappa_2^x - 1}} f(h) \tag{24}$$

---

[26]For example, if flow utility of consumption is linear $\gamma_c = 0$, $\gamma_{pt} > \gamma_{ft}$ and $\epsilon = 0$, the surplus of part-time work over full-time work will be larger smaller for high-skill workers compared to low-skill workers resulting in relatively more part-time jobs in the lower skill segments.

[27]This functional form is similar to those used in Shephard (2017) and Engbom and Moser (2018).

where $A(h, x, p)$ is a term depending on the hiring probability and the discounted expected match duration. The elasticity of vacancy creation with respect to the profit share is $1/(\kappa_2^x - 1)$.

3.2 Estimation Strategy

The remaining structural parameters will be estimated using the simulated method of moments to match important aspects of the German labor market between 2011 and 2014.

Jointly Identified Parameters. The parameters to be jointly estimated are the gender-specific skill distribution parameters (α^g, θ^g), the firm productivity distribution parameters (μ_p, σ_p), the demographic-specific preference parameters (γ_x^j), the type-independent preference parameters $(\gamma_c, \zeta, \varepsilon)$, the search efficiency parameters $(\phi^{su}, \phi^{lu}, \phi^{e_x})$, the vacancy cost parameters (κ_1, κ_2^x), the mass of firms (m_f), the probability of becoming long-term unemployed $(\pi_{lu|su})$, and the labor share of marginal jobs (r_m).

Targeted Moments. To inform these parameters, I target (a) the joint distribution of labor market states and demographics, (b) average and demographic-specific job finding rates out of unemployment, (c) the average elasticity of job finding probabilities with respect to unemployment insurance for short-term unemployed workers, (d) job-to-job transition probabilities conditional on employment level, (e) selected wage quantiles conditional on gender and employment level, (f) the distribution of gender and employment levels in selected wage groups, (g) selected quantile ratios of the gender-specific distributions of worker fixed effects of full-time workers, (h) selected quantile ratios of the distribution of full-time clustered firm fixed effects weighted by the number workers in each employment level, (i) the standard deviation of the log of full-time firm size, and (j) the aggregate job vacancy rate. While all of the parameters are jointly identified by all moments, I will provide intuition for the selection of moments.

In the absence of a minimum wage, the wage equation in my model is very simple. As in Abowd et al. (1999) (henceforth AKM), the wage w of a full-time worker employed at firm with productivity p is log-additive in her skill h and the firm's productivity

$$\log(w) = \log(r) + \log(h) + \log(p) \tag{25}$$

where r is the exogenous piece-rate. I estimate the empirical distribution of worker and firm-class fixed effects using a clustered AKM approach (Bonhomme et al., 2019). In particular, I first cluster firms based on their wage distributions and use firm-class fixed effects instead of firm fixed effects. See Appendix C for details.

To inform the parameters of the skill and productivity distributions, I target selected quantile ratios of the distribution of worker (by gender) and firm fixed effects for full-time workers as well as selected quantile ratios of the distribution of full-time firm fixed effects weighted by the number of part-time and marginal jobs.

Apart from the fixed effects distributions, I target selected quantiles of the gender-specific wage distributions and the overall wage distributions of full-time, part-time and marginal workers. Explicitly targeting the wage distribution is important as the model needs to be able to replicate the pre-reform distribution of wages and employment levels as well as possible.

The search efficiency parameters are closely related to the average job finding probability of short- and long-term unemployment as well as the probability of job-to-job transitions conditional on the current employment level.

The (dis-)utility parameters γ_{ft}^j, γ_{pt}^j and γ_{mj}^j drive heterogeneity in employment status across demographics. The curvature-parameter ζ in the disutilty of job search affects the elasticity of job search with respect to the surplus of employment. Based on the quasi-experimental literature on the UI-elasticity of job finding probabilities I target an average elasticity of 0.5 across all workers (e.g. Chetty, 2008; Schmieder et al., 2012).

The scale parameter κ_1 affects the overall labor market tightness by making vacancies more or less costly and is thus related to the job vacancy rate. The curvature parameters κ_2^x affect the share of type-x jobs across skill-segments and hence across the wage distribution. Increasing κ_2^{mj} relative to κ_2^{ft} will lead to more type-x vacancies in low skill segments as type-x vacancy posting becomes more inelastic with respect to the expected value of vacancy which in turn tends to increase in h. Moreover, decreasing κ_2^{ft} will make it easier for more productive firms to grow large relative to unproductive firms such that the standard deviation of the log of full-time firm size increases. The curvature parameters are thus informed by both the share of part-time and marginal jobs across the wage distribution as well as the standard deviation of the log of full-time firm size.

Numerical Method for Estimation   I estimate the model using a two-step multiple-restart procedure similar to the TikTak-estimation method proposed by Arnoud et al. (2019) and used in Guvenen et al. (2020). In the first stage, I search a compact parameter space by evaluating the objective function at about three million quasi-random Sobol points. I then select the best three thousand points as starting points for local minimizations and pick the local minimizer with the lowest local minimum as the global minimizer.

3.3   Data

The main data source is a 2% sample of administrative social security records of German workers (SIAB) from 2011 to 2014. The SIAB is a linked employer-employee data set containing information on daily earnings and employment levels (full-time, part-time and mini-job) for all German employees that pay social security contributions.[28] Sociodemographic characteristics (apart from gender and age) are only available for nonemployed workers. I thus complement it with survey data from the German Socioeconomic Panel (SOEP) which contains annual information on more than 15 thousand workers. For firm-level moments I use administrative data from the Establishment History Panel and the Job Vacancy Survey of the Institute for Employment Research (IAB) at the German Federal Employment Agency. I focus on prime-aged workers aged 25 to 60. See Appendix C for more details on the data.

3.4   Estimation Results

The model parameters are reported in Table 1 and 2.

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[28]The data does not cover civil servants as they do not pay social security contributions.

TABLE 1: Worker Parameters

Name	Description	Value	Source
All Workers			
β	Discount factor	0.980	–
γ_c	CRRA parameter	0.727	estimated
ζ_2	Search disutility (convexity)	2.056	estimated
ϵ	Relation btw. h and state utilities	0.173	estimated
Skill Distribution of Men			
μ	Mean of $\log(h)$	2.920	estimated
σ	Std. dev. of $\log(h)$	0.542	estimated
Skill Distribution of Women			
μ	Mean of $\log(h)$	2.725	estimated
σ	Std. dev. of $\log(h)$	0.517	estimated
Men, Single			
γ_{ft}^j	State utility of $s = ft$	-0.070	estimated
γ_{pt}^j	State utility of $s = pt$	-0.117	estimated
γ_{mj}^j	State utility of $s = mj$	0.484	estimated
Men, Married			
γ_{ft}^j	State utility of $s = ft$	0.384	estimated
γ_{pt}^j	State utility of $s = pt$	0.130	estimated
γ_{mj}^j	State utility of $s = mj$	0.480	estimated
Women, Single, No Kids			
γ_{ft}^j	State utility of $s = ft$	0.007	estimated
γ_{pt}^j	State utility of $s = pt$	0.226	estimated
γ_{mj}^j	State utility of $s = mj$	0.857	estimated
Women, Single, Kids			
γ_{ft}^j	State utility of $s = ft$	-0.501	estimated
γ_{pt}^j	State utility of $s = pt$	0.531	estimated
γ_{mj}^j	State utility of $s = mj$	0.896	estimated
Women, Married			
γ_{ft}^j	State utility of $s = ft$	-0.210	estimated
γ_{pt}^j	State utility of $s = pt$	0.984	estimated
γ_{mj}^j	State utility of $s = mj$	1.962	estimated

Table 1 shows that the estimated skill distribution of men has a higher mean but lower standard deviation than that of women.[29] Apart from married men, workers receive utility from working fewer hours as $\gamma_f^j < \gamma_p^j < \gamma_m^j$. All women have a higher preference for part-time and marginal jobs. Single women with kids receive the highest disutility from working full-time. The convexity of search cost is close to two. The positive value for ϵ implies that the state (dis-)utilities are scaled up in higher skill segments.

Table 2 shows the firm and labor market parameters. The within-firm relative productivity of part-time and marginal jobs is estimated to be 1.05 and 0.91 respectively.

The vacancy posting cost function for full- and part-time jobs is not very convex as $\kappa_{2f} = 1.75$, $\kappa_{2p} = 1.53$ and $\kappa_{2m} = 2.09$ are not substantially greater than two.[30]

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[29] Figure A.2 in the Appendix A shows the distributions of human capital and firm productivity.

[30] For Brazil, Engbom and Moser (2018) estimate a value of 1.45. Shephard (2017) assumes a quadratic vacancy posting cost function in the UK.
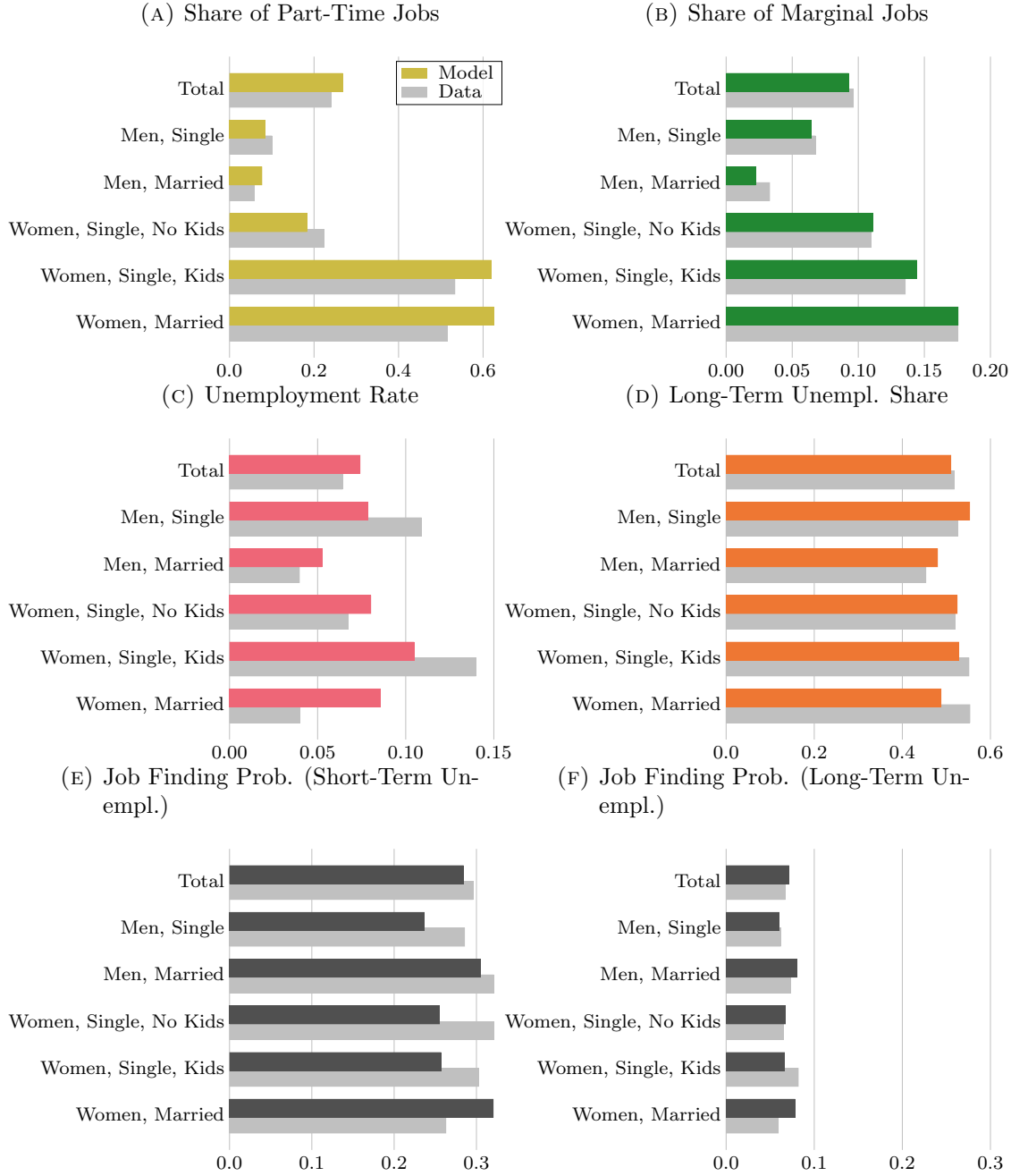
TABLE 2: Firm, Labor Market and Policy Parameters

Name	Description	Value	Source
Firms			
m	Mass of firms	0.025	estimated
α	Scale of $log(p)$	2.269	estimated
θ	Shape of $log(p)$	0.106	estimated
α_{ft}	Relative productivity ($x = ft$)	1.00	normalized
α_{pt}	Relative productivity ($x = pt$)	1.05	estimated
α_{mj}	Relative productivity ($x = mj$)	0.91	estimated
κ_1^{ft}	Vacancy posting cost (weight), $x = ft$	100.0	estimated
$\kappa_1^{pt}/\kappa_1^{ft}$	Relative vacancy posting cost, $x = pt$	0.850	estimated
$\kappa_1^{mj}/\kappa_1^{ft}$	Relative vacancy posting cost, $x = mj$	0.791	estimated
κ_2^{ft}	Vacancy posting cost (convexity), $x = ft$	1.750	estimated
κ_2^{pt}	Vacancy posting cost (convexity), $x = pt$	1.534	estimated
κ_2^{mj}	Vacancy posting cost (convexity), $x = mj$	2.087	estimated
Labor Market			
ξ	Vacancy-elasticity of matches	0.3	P & P (2001)
$\bar{r}_{ft}$	Wage rate ($x = ft$)	0.620	ILO
$\bar{r}_{pt}$	Wage rate ($x = pt$)	0.620	ILO
$\bar{r}_{mj}$	Wage rate ($x = mj$)	0.548	estimated
e_{ft}	Hours ($x = ft$)	1.0	normalized
e_{pt}	Hours ($x = pt$)	0.615	SOEP
e_{mj}	Hours ($x = mj$)	0.223	SOEP
$\pi_{su|e_{ft}}$	Transition from e_{ft} to su	0.010	SIAB
$\pi_{su|e_{pt}}$	Transition from e_{pt} to su	0.019	SIAB
$\pi_{su|e_{mj}}$	Transition from e_{mj} to su	0.030	SIAB
$\pi_{lu|su}$	Transition from su to lu	0.075	estimated
ϕ_{su}	Search efficiency, $s = su$	0.337	estimated
ϕ_{lu}/ϕ_{su}	Relative search efficiency, $s = lu$	0.384	estimated
ϕ_{ft}/ϕ_{su}	Relative search efficiency, $s = e_{ft}$	1.147	estimated
ϕ_{pt}/ϕ_{su}	Relative search efficiency, $s = e_{pt}$	0.911	estimated
ϕ_{mj}/ϕ_{su}	Relative search efficiency, $s = e_{mj}$	0.834	estimated
ψ_{ft}	Godfather shock, $x = ft$	0.017	SIAB
ψ_{pt}	Godfather shock, $x = pt$	0.022	SIAB
ψ_{mj}	Godfather shock, $x = mj$	0.050	SIAB

The top bars in each of the panels of Figure 1 show that the model is able to capture the overall distribution of labor market states and job finding rates.[31] In the estimated model (data), 7.5% (6.4%) of workers are unemployed with 51.4% (51.8%) of them in long-term unemployment. Among the employed workers, 9.0% (9.6%) have a marginal job, 27.4% (24.0%) work part-time and 63.6% (66.3%) have a full-time job. The job finding rate out of short-term unemployment is 28.5% (29.6%) and considerably lower for long-term unemployed workers with 7.0% (6.7%). The difference in job-finding rates reflects the fact that search is estimated to be substantially less efficient in generating matches with firms ($\phi_{lu} < \phi_{su}$). In addition, long-term unemployed workers have lower human capital and thus lower incentives to search for jobs compared to short-term unemployed workers.

The estimated model is also able to capture most of the heterogeneity across demographic groups. Compared to men, a much larger share of women and in particular single women with kids and married women work in part-time or marginal jobs. While the model can replicate

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[31]See table A.4 for the values underlying Figure 1.

FIGURE 1: Model Fit – Employment Moments

(A) Share of Part-Time Jobs

(B) Share of Marginal Jobs

(C) Unemployment Rate

(D) Long-Term Unempl. Share

(E) Job Finding Prob. (Short-Term Un-
empl.)

(F) Job Finding Prob. (Long-Term Un-
empl.)

Notes: This figure shows labor market moments targeted in the estimation for the full population (Total) and within the demographic groups. Subfigures 1 and 2 show the probability of working a part-time and marginal job conditional on being employed. Subfigure 3 shows the unemployment rate and subfigure 4 the share of long-term unemployed workers conditional on being unemployed. Figures 5 and 6 show the job finding probabilities for short- and long-term unemployed workers. Data: SIAB, SOEP.

the observed heterogeneity in employment levels, the unemployment rate of single men and especially single women with kids and married women is less than perfectly matched.

Figure 2 and table A.7 show the distribution of wages over selected wage bins. The overall fit (panel A) is remarkably good given the limited flexibility imposed by the parametric skill and productivity distributions and the fact that there are no skill-dependent parameters.[32] Only 2.4% (1.8% in the data) of all jobs pay a wage below €6.5, 8.5% (9.8%) of wages are above €6.5 but below €8.5, 22.1% (18.8%) of wages are between €8.5 and €12.5, 33.6% (34.6%) are between €12.5 and €20 and 33.4% (35.0%) of wages exceed €20. The model is also able to capture gender-specific heterogeneity as a larger share of women find themselves in the lower wage bins. Similar to the data, 14.1% (16.5%) of women are affected by the initial minimum wage, only 7.8% (6.7%) of men earn less than €8.5 per hour.

The differences in the job-type-specific wage distribution (panels B to D) are also replicated by the model. Full-time jobs pay substantially higher wages than part-time jobs, which in turn pay higher wages than marginal jobs. Hence, minimum wages will cut deeper into the wage distribution of part-time and marginal jobs compared to full-time jobs. In particular, the initial minimum wage affects 45.8% (53.9%) of marginal jobs, 10.8% (12.1%) of part-time jobs but only 5.8% (5.5%) of full-time jobs. The most important difference between model and data is that the distribution of wages for marginal jobs is too dispersed. There are too many jobs paying a wage below €6.5 or above €12.5 and too few jobs in the range between €6.5 and €12.5. In addition, too few full-time jobs pay wages between €8.5 and €12.5. This will affect how the distribution of job types is affected by the minimum wage. Figure 3 shows the share of full-time, part-time, marginal jobs and men in each of these wage bins. Marginal jobs are over-represented in the lowest wage bin. In addition, part-time jobs are over-represented in the wage bins around the initial minimum wage of €8.5 as there are not enough full-time jobs in this range. While these differences between model and data need to be kept in mind, the model delivers a good fit to the joint distribution of wages and job types.

The distribution of worker and firm fixed effects for full-time jobs is shown in Figure 4.

Figure 5 shows the distribution of full-time firm fixed effects among part-time and marginal jobs. In particular, panels C and D show the employment weighted variation in firm productivity among part-time and marginal jobs which the model is able to match quite closely. Panel E shows the percent difference between the qth quantile of the firm productivity distribution weighted by part-time employment and the corresponding quantile of the firm productivity distribution weighted by full-time employment. Both in the data and the model, firm productivity is just slightly lower among full-time workers (about 5%). Using marginal workers as weights instead of full-time workers, the firm productivity distribution shifts downward by around 20% in the data but by significantly more in the model (panel F). Hence, marginal workers in the model work at firms that pay too low full-time wages compared to the data.[33]

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[32]Engbom and Moser (2018) estimate a set of labor market parameters for each skill segment.
[33]See Table A.8 and Table A.9 for details.

FIGURE 2: Model Fit – Wage Groups by Job Types and Gender

(A) Total

(B) Full-Time Jobs

(C) Part-Time Jobs

(D) Marginal Jobs

(E) Men

(F) Women
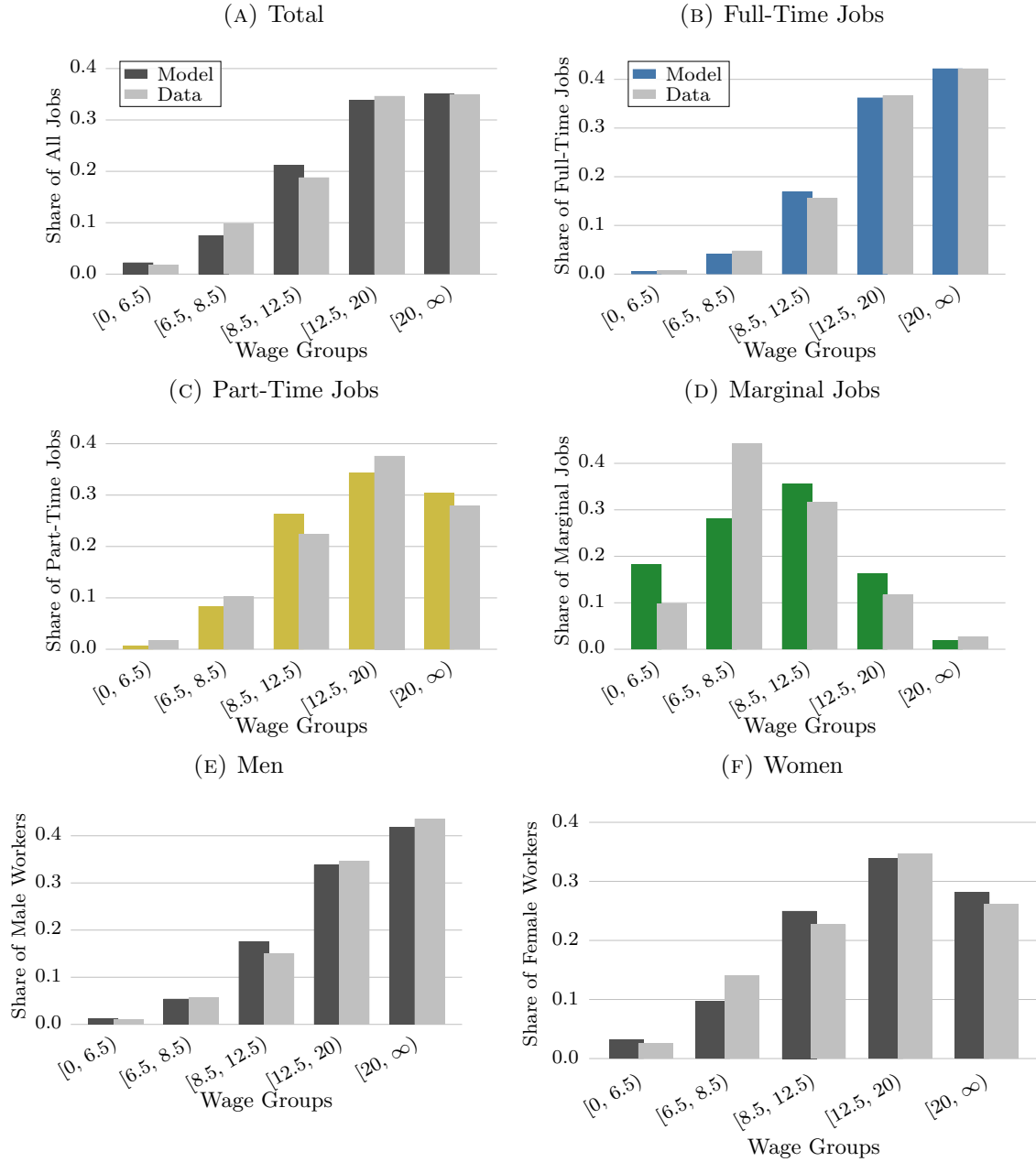
Notes: This figure shows the distribution of jobs over four wage groups for allworkers and separately for full-time, part-time, marginal job, male and female workers in the model and data. Data: SIAB, SOEP.

FIGURE 3: Model Fit – Job Types and Gender By Wage Groups

(A) Full-Time Jobs

(B) Part-Time Jobs

(C) Marginal Jobs

(D) Men
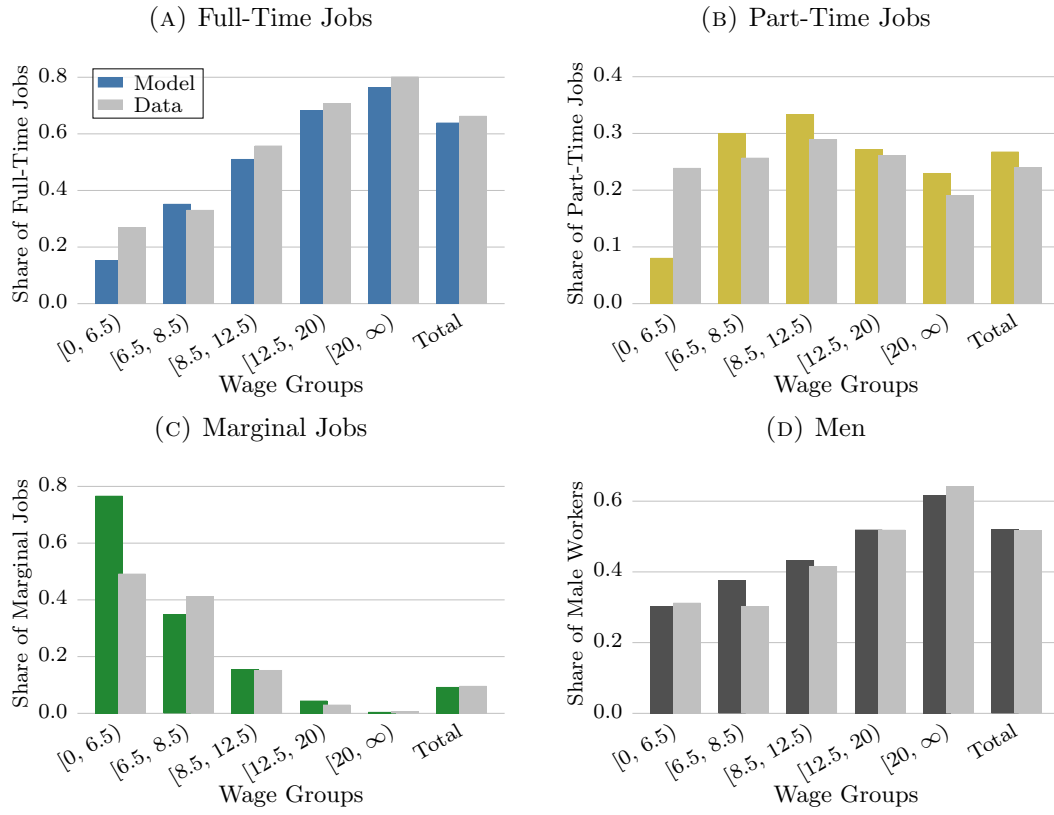
Note: This figure shows the share of full-time, part-time and marginal jobs as well as the share of men within various bins of the wage distribution in the model and data. Data: SIAB, SOEP.

FIGURE 4: Model Fit – Worker Fixed Effects

(A) Men
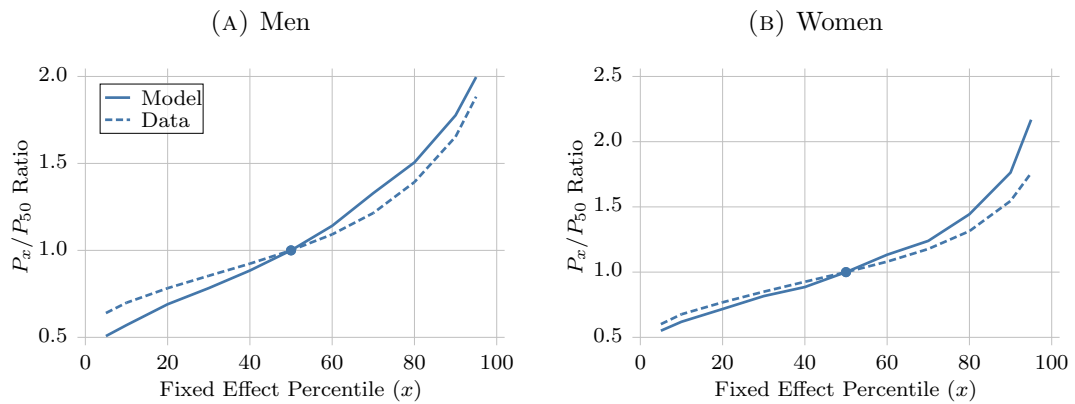
(B) Women

Note: This figure shows the ratios of selected percentiles to the median of the distributions clustered AKM worker fixed effects for men and women. See appendix C for details. Data: SIAB.

FIGURE 5: Model Fit – Firm Fixed Effect

(A) Total

(B) Full-Time

(C) Part-Time

(D) Marginal

(E) Part-Time / Full-Time

(F) Marginal / Full-Time
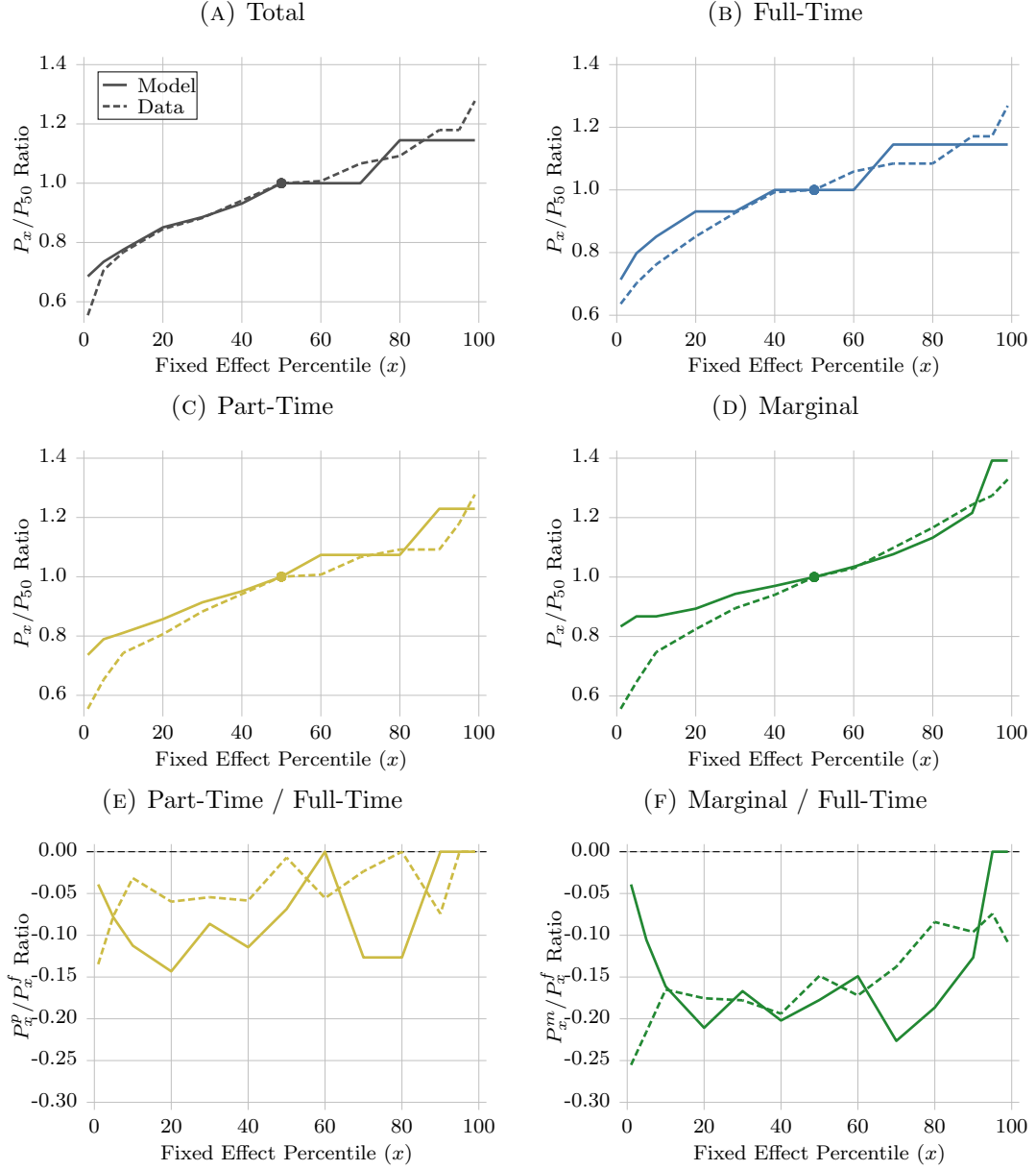
Note: This figure shows the distribution of (clustered) firm fixed effects estimated using clustered AKM on full-time jobs. In panels A, B, C and D, all jobs, only full-time, only part-time jobs and only marginal jobs are used as weights respectively. Panels E and F show how the distributions change when weighting by part-time and marginal jobs instead of full-time jobs. Data: SIAB.

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4 The German Minimum Wage Reform of 2015

In 2015, the German government introduced a federal minimum wage of €8.5 (Kaitz index of 47%) that cut deep into the wage distribution affecting more than 10% of all jobs. In this section, I use the estimated model to analyze how the initial federal minimum wage affected employment, productivity and output. I first compare the pre- and post-reform steady states and highlight the mechanisms at play (Section 4.1). I then analyze the transitional dynamics (Section 4.2) and compare the predicted short-term effects to the available reduced form evidence (Section 4.3)

4.1 Steady State Comparison

Table 3 compares the steady state without a minimum wage to the steady state with a minimum wage of €8.5 (column 2). The difference between the new and the old stationary equilibrium is shown in column 3.

Relative to the steady state without a minimum wage, the unemployment rate is slightly lower (0.035 percentage points) in the steady state with a minimum wage of €8.5. The small change in the number of jobs masks heterogeneity across employment levels. In particular, while the share of marginal jobs among all jobs drops from 9.14% to 7.94%, the share of part-time and full-time jobs increases by 0.81 and 0.39 percentage points respectively.

The slight decrease in the unemployment rate occurs despite a small drop in the average job finding rate out of unemployment, $\Pr(e|u)$, by 0.124 percentage points (0.07%). This is largely driven by a change in the composition of the unemployed. With the minimum wage in place, the share of long-term unemployed and low-skill workers among all unemployed workers increases and pushes down the average job finding rate of unemployment. Similarly, the average job destruction probability falls by about 0.02 percentage points (1.2%) because of reallocation away from relatively unstable marginal jobs towards more stable part-time and full-time jobs.[34]

Average wages in the new stationary equilibrium are up by about 2.1%. Part of this increase is driven by reallocation to more productive firms. In other words, workers now work at firms where they would have received 0.5% higher wages even in the absence of a minimum wage. While over two thirds of the increase in productivity reflects reallocation to more productive firms (higher p), part of the increase in productivity ($a_x p$) is a result of the shift away from relatively unproductive marginal jobs.

Average gross earnings increase by more than wages (+3.5%) reflecting the shift towards jobs with longer hours (+1.4%). Taxes and transfers result in a 2.8% increase in average earnings and a 0.8% increase in incomes. The relatively weak increase in incomes follows from the fact that many low-skill workers top up their earnings with unemployment benefits. Reallocation to better firms and longer hours leads total output to grow by 0.5%. While the tax-and-transfer scheme mutes the increase in incomes, total transfer payments decrease by 6.0%. In addition, the government's revenues from labor taxation increase by 0.9% as average earnings grow and the unemployment rate falls slightly.

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[34]Recall that the job destruction probability of any given employment relationship is exogenous and thus not affected by the minimum wage. Endogenous job destruction only occurs if a job becomes unprofitable due to a prohibitively high minimum wage. In the new steady states, however, these jobs are not created in the first place. The drop in the average job destruction rate is hence a pure composition effect.

TABLE 3: Minimum Wage Effects – General Equilibrium

	(1)	(2)	(3)
	Baseline ($\bar{w}=0$)	New Equilibrium ($\bar{w}=8.5$)	
	Value	Value	Change
Labor Market States			
Unemployment Rate	7.42%	7.39%	-0.034
Long-Term Share	51.07%	51.26%	0.196
Full-Time Share	63.94%	64.38%	0.435
Part-Time Share	26.79%	27.59%	0.803
Marginal Share	9.27%	8.03%	-1.238
Transition Probabilities			
$\Pr(e|u)$	17.57%	17.43%	-0.137
$\Pr(su|e)$	1.41%	1.39%	-0.018
Wages, Earnings & Incomes			
Mean Log Wages	2.797	2.817	0.020
Mean Log Productivity (p)	0.376	0.381	0.004
Mean Log Productivity ($\alpha_x p$)	0.382	0.388	0.006
Mean Log Hours	3.389	3.404	0.015
Mean Log Earnings	7.652	7.687	0.035
Mean Log Net Earnings	7.297	7.326	0.028
Mean Log Income	7.598	7.605	0.008
Macro Aggregates			
Log Output	8.305	8.311	0.005
Log Transfers	4.549	4.489	-0.059
Log Labor Taxes	6.750	6.759	0.009

Note: This table shows the long-run general equilibrium effects of the introduction of a federal minimum wage of 8.5 EUR relative to the baseline equilibrium without a minimum wage (first column). Changes refer to the absolute difference to the baseline outcome (e.g. percentage points or log points).

In sum, the introduction of the minimum wage introduction moves the economy into an equilibrium with higher productivity, output and employment. While the unemployment rate decreases only slightly, employment weighted by hours worked increases markedly as the share of part-time and full-time jobs rises. The minimum wage thus partly removes misallocation towards low-earnings jobs created by search frictions and transfers to workers with low-earnings jobs. While the tax-and-transfer system prevents incomes from growing more strongly, workers are less reliant on government transfers to top up their earnings. Combined with the fact that higher average earnings raise tax revenues, the reform improves the government's budget position.

Mechanisms I now study the importance of the different mechanisms that feed into the general equilibrium effects. To that end, I shut down different margins of adjustment one at a time. Table A.11 shows the partial equilibrium effects of fixing fix workers' search effort, workers' surplus of successful search, firms' vacancy posting, the vacancy shares, and the mass of vacancies to the baseline levels in columns 3 through 7. Columns 1 and 2 report the baseline levels and general equilibrium effects from Table 3. Figure 6 visualizes the general and partial equilibrium effects for unemployment, output, hours worked and firm productivity.

Panel A of Figure 6 shows that eliminating workers' job-finding surplus or search effort pushes the unemployment rate up while shutting down firms' vacancy posting pushes it down. When

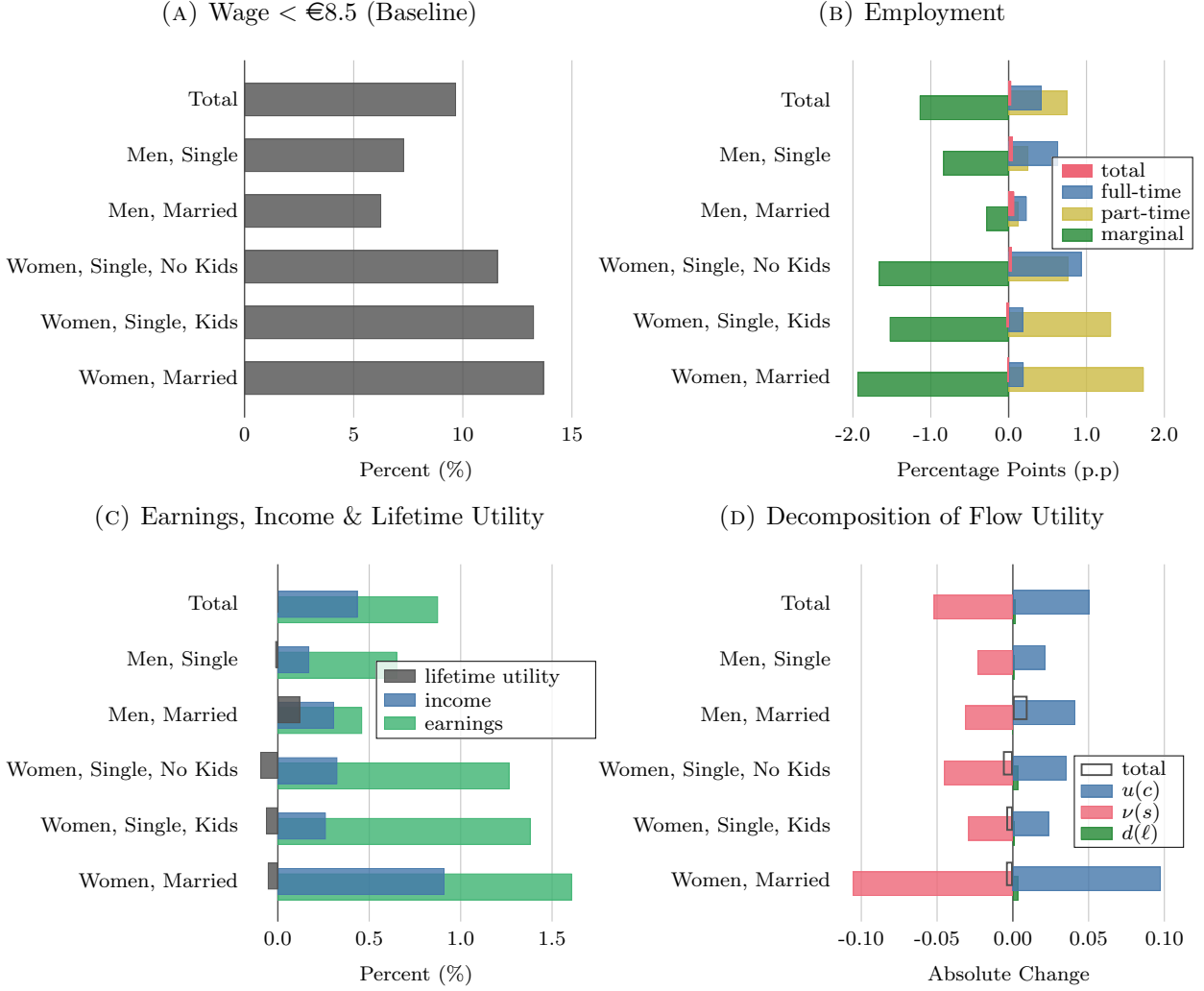FIGURE 6: Mechanisms - Partial vs. General Equilibrium

(A) Unemployment Rate

(B) Output

(C) Average Hours Worked

(D) Average Productivity
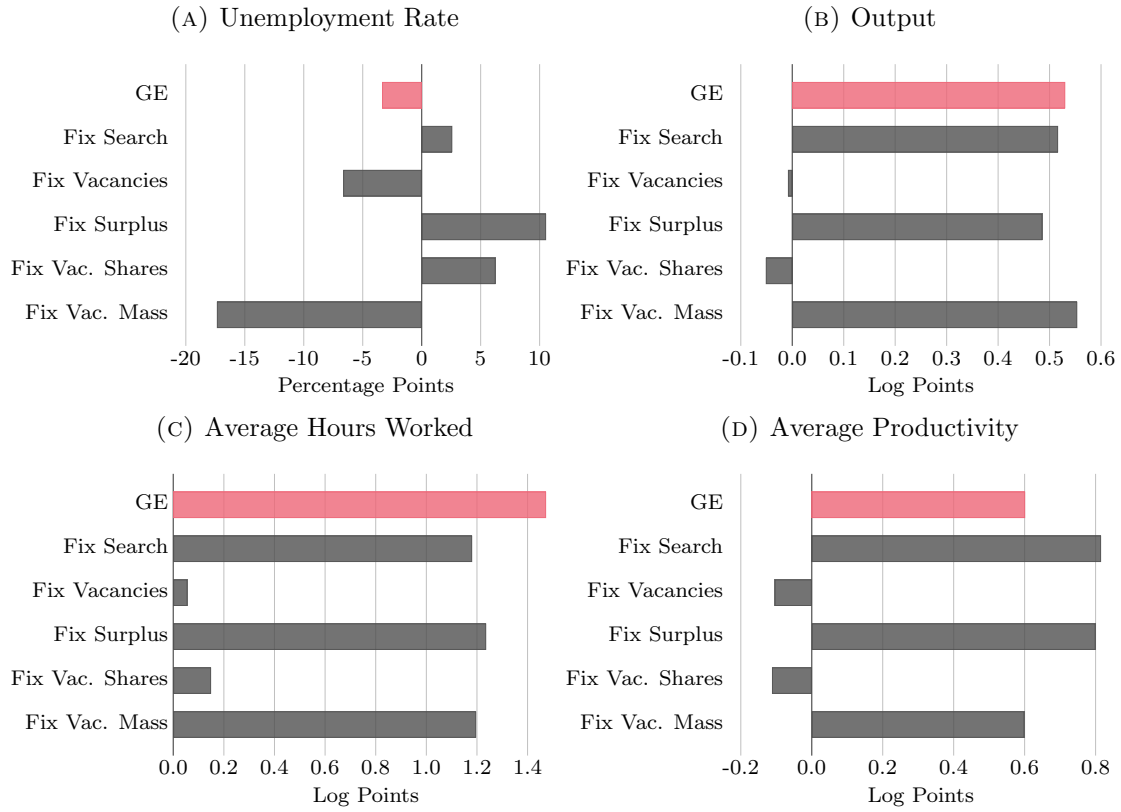
Notes: This figure shows the effects of the minimum wage of €8.5 for different partial equilibrium scenarios that result from shutting down different margins of adjustment. If a partial equilibrium differs from the general equilibrium scenario, the respective channel is important for generating the general equilibrium effect.

workers' surplus of successful search is held fixed, the unemployment rate increases by about 0.13 percentage points instead of the decrease by 0.035 percentage points in general equilibrium. The effect of fixing search effort is smaller than that of fixing the surplus since search effort is negatively affected by the drop in labor market tightness. When firms' vacancy posting policies are held fixed, the unemployment rate decreases by 0.9 percentage points. Taking a closer look at the role of vacancy posting, we see that there are two opposing effects. On the one hand, the total mass of vacancies is reduced, which drives up unemployment (via lower job finding rates). On the other hand, the change in the composition of posted vacancies away from unstable low-hours jobs lowers unemployment (by reducing the average job destruction rate). Besides this effect on average job destruction rates, the change in the hours-distribution of vacancies raises searchers' expected disutility from longer working hours and thus dampens the increase in the surplus of successful search and hence search effort and job finding rates. The reduction in overall vacancy posting, however, dominates so that the net effect of endogenous vacancy posting on the unemployment rate is positive.

The increase in average hours worked (panel C) and firm productivity (panel D) is driven mainly by firms' vacancy posting and in particular by the change in the composition of vacancies. In general, firms create fewer vacancies for jobs that are (strongly) affected by the minimum wage. As the minimum wage affects low-hours and low-productivity jobs relatively often, the

FIGURE 7: Heterogeneous Effects by Sociodemographics

(A) Wage < €8.5 (Baseline)

(B) Employment

(C) Earnings, Income & Lifetime Utility

(D) Decomposition of Flow Utility

Notes: This figure shows how the effects of the minimum wage of €8.5 vary across demographic groups. Panel A shows how many employed workers are affected by the minimum wage, panel B shows how the distribution of labor market states changes (the bars sum to zero). Panel C shows the relative change in average earnings, income/consumption and lifetime utility. Panel D decomposes the average change in flow utility into its components (see equation 22).

reduction in vacancies is not symmetric across employment levels. Conditional on meeting a firm, the probability of being offered a low-hours or low-productivity job declines.

For total output (panel B), I find that the reallocation effect is much more important than changes in the number of jobs. Only fixing the mass of vacancies – which has a relatively big effect on the unemployment rate – does not lead to a lower output effect. The distribution of vacancies across employment levels and firm productivity drives the positive output effect in general equilibrium.

Heterogeneity Across Sociodemographics The different demographic groups in the model and the data are differently affected by the minimum wage. Figure 7 shows that women are significantly more likely to earn less than €8.5 per hour. I now analyze how the effects of the reform vary across demographic characteristics in the new stationary equilibrium.

Panel B of Figure 7 displays the percentage point changes in the distribution of labor market states (full-time, part-time, marginal and total employment). All bars sum to zero. While the reallocation pattern away from marginals towards part-time and full-time jobs is the same qualitatively, there is substantial variation in magnitude. en and single women without kids move to both part-time and full-time jobs. In contrast, the share of married women and single women wit kids working full-time jobs hardly increases because of the high disutility of working full-time for this group. As a result, their unemployment rate increases slightly while total unemployment drops.

Panel C shows how lifetime utility, income and earnings change relative to the baseline equilibrium. Although earnings increase substantially, income growth is much weaker due to the fact that many low-wage workers top up their earnings with government transfers and thus lose the majority of the earnings increase. Perhaps surprisingly, lifetime utility remains almost unchanged and is slightly negative for women. This is because the small increase in income (consumption) is counteracted by lower state utility as workers now work longer hours. Especially those workers who have a strong preference for or rely on marginal jobs with low working hours experience utility losses from the reallocation towards part- and full-time jobs.

To see this more clearly, panel D decomposes the average change in flow utility (closely correlated with lifetime utility) into the components of the utility function. While utility from consumption, $u(c)$, increases, hours-related utility, $\nu(s)$, decreases. Disutility from search plays almost no role.

4.2 Transitional Dynamics

In the presence of search frictions, the process of worker reallocation takes time. Workers whose jobs survive the introduction of the minimum wage will gradually transition to more productive firms or jobs with longer hours. More importantly, the minimum wage will make some jobs unprofitable. These workers become unemployed and finding a (better) job takes time. While worker reallocation pushes up output in the long-run, the short-run effects of introducing or raising the minimum wage may be significantly less desirable. It is thus paramount to study the transitional dynamics triggered by the minimum wage reform.[35]

Figure 8 shows how the economy reacts to the reform. Panel A shows that there is indeed a drop in total employment as some jobs become unprofitable. It takes about five years until the employment response turns positive. The matgnitude of the initial layoff shock, however, is very small (0.052 percentage points). It takes roughly ten years for the shift towards part-time and full-time jobs to unfold, but the majority of the shift occurs in the first five years.

Driven by this increase in average hours worked, panel B shows that while both average wages and earnings jump up immediately after the reform, earnings continue to grow substantially over the following ten years.

Panel C decomposes the growth in average wages. Initially, wage growth is almost entirely driven by lower profit margins and thus a higher average labor share. As time progresses, workers

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[35]Appendix D explains how the transition path is computed.

FIGURE 8: Dynamic Effects of the Initial Minimum Wage

(A) Employment

(B) Wages, Hours and Earnings

(C) Log Wage Decomposition
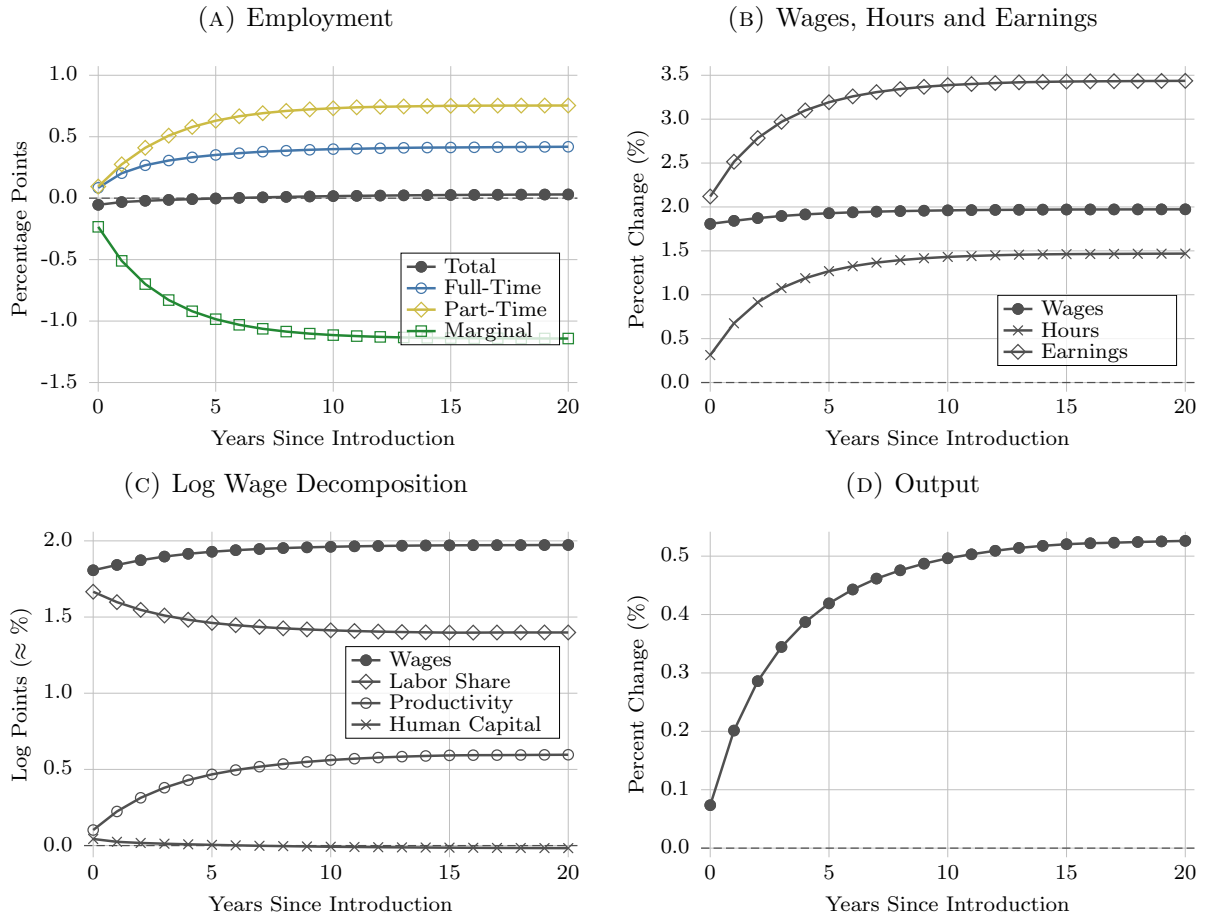
(D) Output

Notes: This figure shows the predicted changes in employment by job type (panel A), averages wages, hours and earnings (panel B), the components of wage growth (panel C), and total output (panel D) following the introduction of a minimum wage of €8.5.

reallocate to more productive firms. Hence the profit margin recovers and workers' higher wages are increasingly the result of working for more productive firms.[36]

Finally, panel D shows that total output increases monotonically following the reform since the initial dip in employment is negligible. After five years, total output is already 0.39 log points above the pre-reform level (80% of the overall effect).

4.3 Comparison with Reduced-Form Evidence

Using these transition results, I now briefly discuss how the model predictions line up with the available reduced-form evidence on the initial introduction of the minimum wage which can be seen as an independent test of the model. There are several studies documenting the short-run effects of the 2015 minimum wage reform using individual or regional variation in the bite of the minimum wage (e.g. Garloff, 2016; Caliendo et al., 2017; Holtemöller and Pohle, 2017; Burauel et al., 2020; Dustmann et al., 2022). The results of these studies boil down to the following points.

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[36]We also see that wage growth is not driven by selection of relatively high-skill workers into employment.

First, hourly wages increased significantly and consistent with near full compliance from 2014 to 2016. According to (Dustmann et al., 2022), average wages grew by about 1.5% compared to 2.0% in the model.[37] Importantly, the authors show that, consistent with the model (Figure 8), earnings grew significantly more than wages (2.4% in the data and 2.8% in the model).

Second, and again consistent with the model, none of the afore-mentioned studies find significant adverse effects on overall employment.[38] However, the minimum wage induced a shift from marginal jobs towards part-time and full-time jobs (Garloff, 2016; Holtemöller and Pohle, 2017). Caliendo et al. (2017) estimate that approximately 2.4% of marginal jobs were lost due to the minimum wage in the first year of the reform. vom Berge et al. (2016) document that the number of marginal workers dropped by about 2% and 4% from December 2014 to January and September 2015 respectively.[39] The model predicts that about 2.7% of marginal jobs were lost on impact and about 4.4% in the fall of 2015.

Third, there is also evidence that the minimum wage reallocated workers to larger, more productive firms. (Dustmann et al., 2022) attribute 17% of the increase in wages from 2014 to 2016 to reallocation towards more productive firms. In the model, 15% of the wage gain over the first two years comes from reallocation.

In sum, the estimated model captures all these effects qualitatively and does a good job of replicating them quantitatively. The fact that the model not only matches well the labor market moments in the pre-reform period, but is also broadly consistent with the rich reduced-form evidence on the minimum wage reform lends credibility to the optimal policy analysis in section 5.

5   Counterfactually High Minimum Wages

In this section, I use the model to analyze how counterfactually high minimum wage levels will impact employment, output and worker welfare. First, I will analyze the long-run employment and output effects (Section 5.1). Second, I will analyze the entire transition path and compare short- and long-run effects (Section 5.2). Third, I will discuss heterogeneity in welfare effects by worker skill and demographics (Section 5.3).
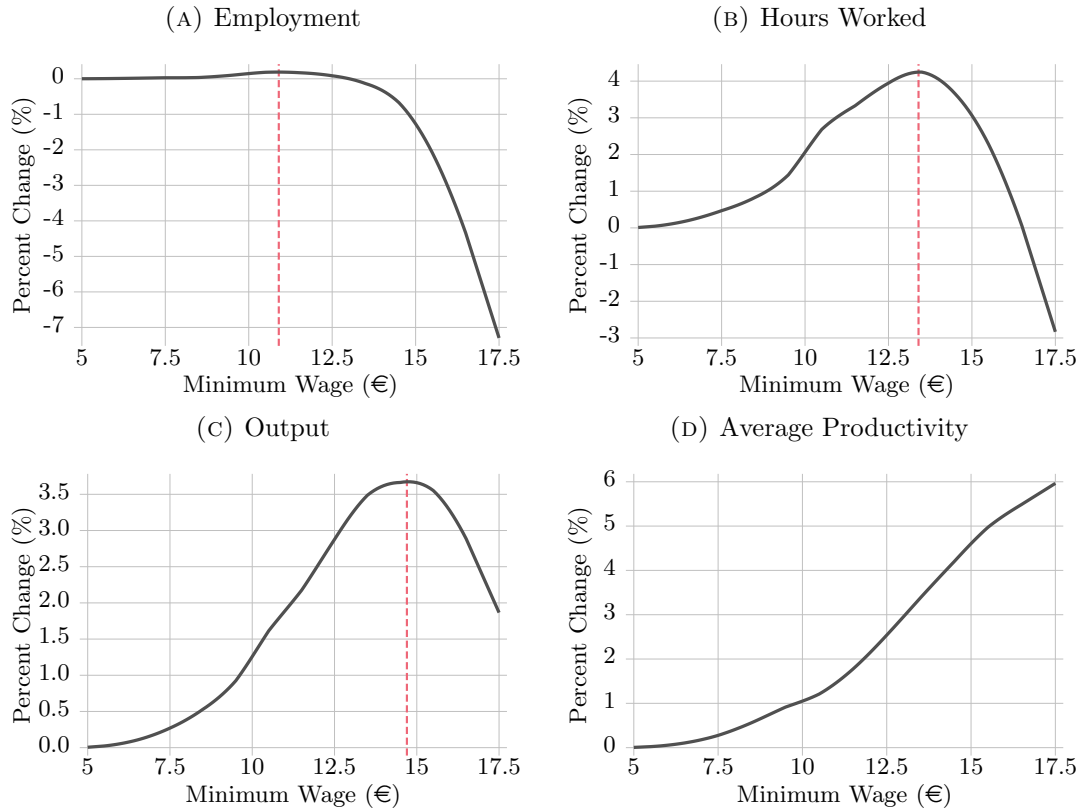
5.1   Long-Run Effects

I first take a long-run perspective by comparing the stationary equilibrium that emerges for different minimum wages to the baseline equilibrium. Figure 9 shows steady-state employment, total hours worked, total output and average productivity as a function of the minimum wage. Panel A shows that total employment, i.e. the share of employed workers, is a non-monotonic function of the minimum wage. Employment is maximized at a minimum wage of €11.0 (Kaitz

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[37]While the discrepency between model and data is small (and not targeted), the difference is explained by small differences in the initial wage distributions between the model and the data (Figure 2). In particular, the model's wage distribution features more workers with wages below €6.5 and fewer workers with wages between €6.5 and €8.5 compared to the data. Hence, average wage growth under full compliance is slightly higher in the model. The numbers from Dustmann et al. (2022) refer to their individual-level analysis (Table 2).

[38]In addition and consistent with the model, turnover rates decreased as both job finding and separation rates were reduced (Bossler and Gerner, 2016).

[39]Caliendo et al. (2017) include all marginal workers while vom Berge et al. (2016) only include those workers where the marginal job is the main job. Note that I use the same definition as vom Berge et al. (2016).

FIGURE 9: Long-Run Minimum Wage Effects

(A) Employment   (B) Hours Worked

(C) Output   (D) Average Productivity

Notes: This figure shows the predicted long-run minimum wage effects on employment, hours worked, output and average productivity in panels A through D respectively. The red dashed lines in panels A, B and C indicate the maximum.

= 60%) and does not drop below the baseline level for minimum wages below €13.0 (Kaitz = 70%). Quantitatively, the positive employment effect of moderate minimum wages is very small (≤ 0.2 p.p.) while the decline in employment for high minimum wages is quite steep.

Panel B shows that the minimum wage not only affects the extensive margin of employment but also the average employment level. While the number of employed workers is essentially flat for moderate minimum wages, total hours worked increase significantly. Hence, the hours-maximizing minimum wage of €13.5 (Kaitz = 73%) is considerably higher than the employment-maximizing minimum wage. Even when employment starts to decline, hours worked continue to increase driven by the intensive employment margin.

This increase in total hours worked implies that the output-maximizing minimum wage is also significantly higher than the employment-maximizing one. Indeed, Panel C shows that output increases considerably in the minimum wage up until €14.4 (Kaitz = 78%). At the optimum, total output is 3.7% higher compared to the baseline without a minimum wage. The increase in total output is not only driven by the increase in hours worked. Panel D shows that *average* firm productivity increases monotonically in the minimum wage. This second margin of reallocation pushes the output-optimum above the hours-optimum. At the output-maximizing minimum wage, total hours worked are 4.3% above the baseline level and average firm-productivity is up by 4.1%.

FIGURE 10: Long-Run Minimum Wage Effects – Mechanisms

(A) Employment

(B) Hours Worked

(C) Output

(D) Average Productivity
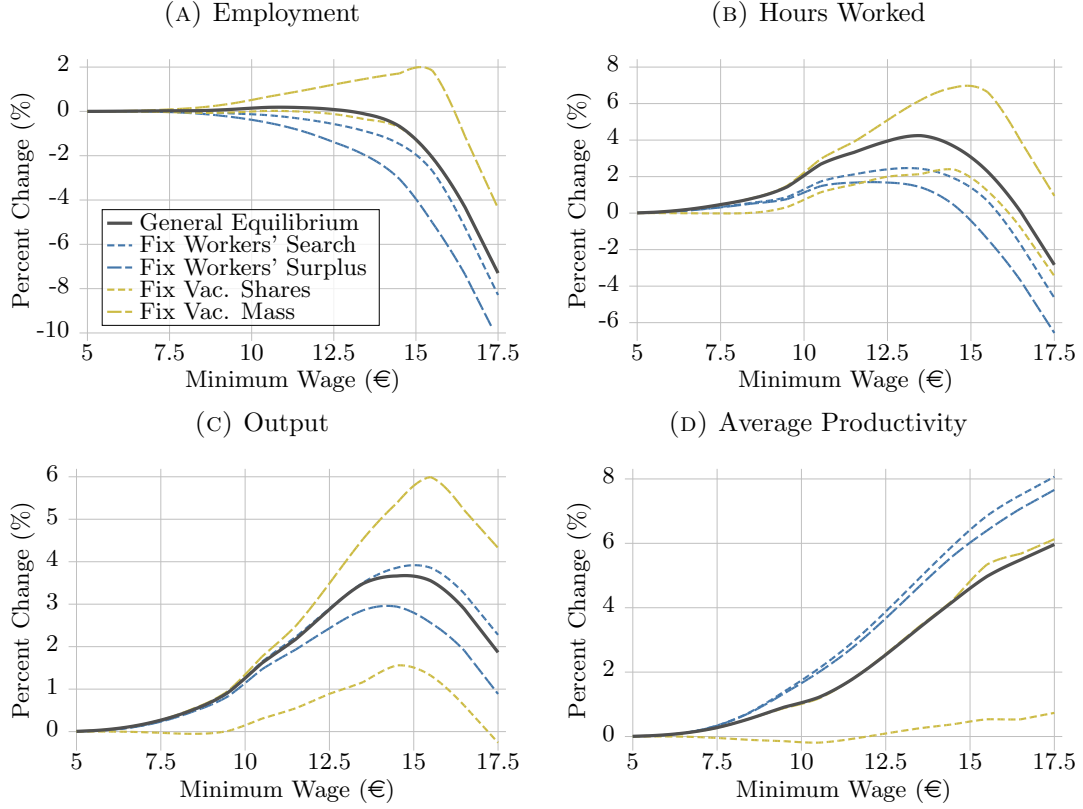
Notes: This figure shows the predicted relative changes in total employment, total hours worked, total output and average productivity as a function of the minimum wage for different scenarios. The blue short-dashed line shows a scenario where workers' search effort is held fixed at the pre-reform levels. The blue long-dashed line refers to a scenario that allows search effort to adjust, but keeps the surplus of meeting a firm constant. The yellow short-dashed line displays a scenario where the hours-and productivity distribution of vacancies is held fixed while the total mass of vacancies is allowed to adjust. The yellow long-dashed line refers to a scenario where the distribution of vacancies is flexible but the mass of vacancies is fixed.

In order to understand what drives these minimum wage effects in Figure 9, I fix (a) workers' search policies, (b) the expected surplus of meeting a firm, (c) the productivity-hours distribution of vacancies, and (d) the mass of vacancies at the corresponding baseline levels. Figure 10 illustrates the results in each of these partial equilibrium scenarios.

Panel A shows that endogenous search effort and, in particular, the increase in the expected surplus of meeting a firm is responsible for the lack of disemployment effects for moderate minimum wages in general equilibrium. If workers' are not allowed to adjust their search behavior, employment decreases monotonically as the minimum wage exceeds €7.5. If workers are allowed to re-optimize, but the surplus of contacting a firm is held at its baseline level, the drop in employment is even more pronounced. This is because workers now reduce their search effort as firms post fewer vacancies and the aggregate contact rate drops (search effectiveness). In contrast, when the number of vacancies is held constant and workers' can adjust their search effort, total employment increases significantly in the minimum wage. The negligible employment effects for moderate minimum wages is therefore not due to a muted reduction of firms' vacancy posting, but rather the net effect of two off-setting forces. In other words, changes in the demand and supply of labor largely offset each other for moderate minimum wage hikes. As

the minimum wage approaches €15, however, broad non-employability of low-skill workers kicks in and employment falls.[40]

Whereas changes in the productivity and employment level distribution of vacancies have only a small impact on total employment, they drive the response of both total hours worked and average productivity (Panels B and D). It is clear that the profitability of low-productivity jobs declines relative to that of high-productivity jobs for a given minimum wage. Asymmetric declines in profitability lead to asymmetric vacancy reductions.[41] The minimum wage shifts the productivity distribution of vacancies to the right. Conditional on meeting a firm, average firm productivity thus increases and workers move to more productive firms. Panel D shows that fixing the composition of vacancies kills the positive productivity effect of minimum wages.

An important feature of the data and the model is that full-time vacancies are offered by relatively more productive firms than vacancies for part-time and especially marginal jobs. Hence, the response of firms' vacancy posting is not only asymmetric in terms of productivity but also in terms of employment levels. Therefore, fixing the composition of vacancies also mutes the effect on total hours worked as there is less reallocation towards high-hours jobs.

While firms' vacancy posting decisions drive up hours worked and average firm productivity following a minimum wage hike, the optimal response of workers increases hours worked but decreases average firm productivity. On the one hand, a binding minimum wage increases the relative value of high-hours jobs and thus increases the incentives of marginal and part-time workers to engage in on-the-job search for a job with a higher employment level. This is because, for a given increase in the hourly wage, earnings and therefore consumption growth is higher for jobs with higher employment levels. Therefore, fixing fixing workers' search effort or their surplus of meeting a firm at the baseline levels reduces the positive hours effect of minimum wages by about 50%. Conversely, a binding minimum wage reduces the surplus of working for a high productivity firm as the minimum wage reduces productivity-related wage differentials. This reduces the incentives for on-the-job search and the probability that a worker at a low-productivity firm will accept a job offer from a high-productivity firm (with the same employment level). Hence, allowing workers' to adjust their search effort has a muted effect on total output (Panel C) as it pushes hours worked and average firm productivity in opposite directions.

In sum, the comparisons of steady states show that increasing the minimum wage creates a trade-off between employment and output. Policy makers can use the minimum wage to improve the average productivity and employment level of jobs and thereby reduce misallocation arising from search frictions. However, the model predicts that, for minimum wages beyond a Kaitz index of 60%, improved job composition has to be traded off against lower total employment.

In Figures A.4 and A.5, I show how this result depends on the assumptions regarding two key parameters, the vacancy-elasticity of the matching function ξ and the piece rate r workers receive. In particular, I show the long-run employment and output effects as a function of the minimum wage's Kaitz index for alternative values of ξ and r. As expected, if the number of matches is more (less) responsive to a change in vacancies or if the piece rate is higher (lower),

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[40]Note that in the scenario where the mass of vacancies is held fixed, I exclude non-employable vacancies.

[41]This can be seen in Figure A.3, which plots the equilibrium productivity distribution of vacancies for selected minimum wages.

FIGURE 11: Initial Job Destruction
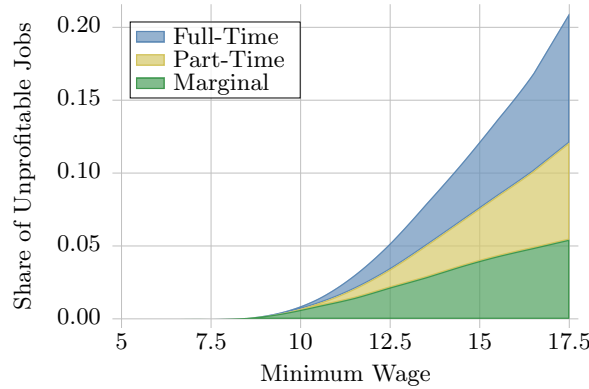
Notes: This figure shows what share of jobs in the baseline equilibrium becomes unprofitable for different minimum wages. The different areas decompose the total share into different employment levels.

employment starts to drop sooner (later). Importantly, the general insight that higher minimum wages (Kaitz index above 60%) can yield significant output gains through reallocation at the expense of limited employment losses is very robust.[42]

5.2 Transition Dynamics: Long-Run Gain vs. Short-Run Pain

The steady state comparisons shows that reallocation is crucial to understand the effects increasing the minimum wage. While search frictions are the fundamental reason why the minimum wage can improve production efficiency, they also imply that any reallocation process takes time and may be quite painful. Taking search frictions seriously thus requires analyzing the entire transition path.

Figure 11 shows how many jobs in the baseline equilibrium will become unprofitable for different minimum wages. The higher the minimum wage, the more jobs will be destroyed following the minimum wage hike. While initial job destruction is not important for minimum wages below €10, it is increasingly relevant for higher minimum wages. At the long-run output maximum of €14.4, for example, over 10% of all jobs are destroyed initially. These workers become unemployed and have to find a new (better) job which takes time and effort.

Figure 12 shows how the unemployment rate and output evolve following minimum wage hikes of different magnitudes. In particular, the black line corresponds to the observed minimum wage reform (€8.5) and the green, yellow and red lines correspond to the long-run employment maximum (€11), the highest minimum wage without long-run disemployment effects (€13.0), and the long-run output-maximizing minimum wage (€14.4) respectively.

As expected, we see significant spikes in the unemployment rate at the time the minimum wage is imposed (panel A). At the long-run employment-maximizing minimum wage of €11, the unemployment rate increases by about 1.5 percentage points on impact (increase of 20%). While a minimum wage of €13.0 does not lead to disemployment effects in the long-run, it does so in the short- and medium run as the economy takes about 10 to 15 years to convergence to

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[42]The fact that the maximal output gain from reallocation varies is largely due to the changes in the initial equilibrium, which, for higher values of ξ or r features more full-time jobs and thus less room for reallocation along that margin.

FIGURE 12: Minimum Wage Effects Along the Transition Path
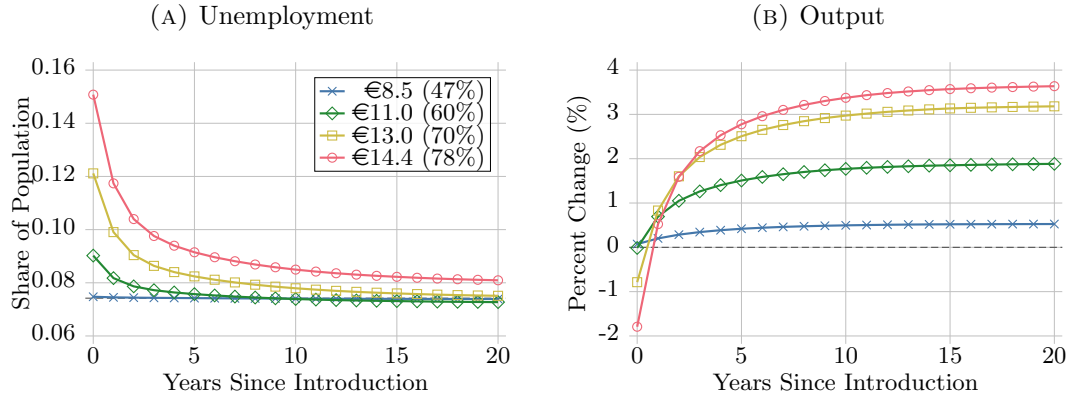
(A) Unemployment

(B) Output

Notes: This figure shows how the unemployment rate and output evolve following minimum wage hikes of different magnitude – always starting at the baseline equilibrium without a minimum wage. I assume that the minimum wage hike is announced one quarter before it becomes binding.

the new stationary equilibrium. At the long-run output-maximizing minimum wage of €14.4, the unemployment rate more than doubles following the reform and takes three years to fall below 10%. After five years, the unemployment rate is still 24% above the baseline and 15% above the new long-run unemployment rate.

Panel B shows that output gains also take time to materialize. For minimum wages above €11, the minimum wage hike forces the economy into a recession. On the way to the long-run output maximum, output is below its baseline level for almost two years. Nevertheless, output gains take less time to kick in than it takes the unemployment rate to drop. This is because high short-run unemployment is mostly driven by workers at the bottom end of the skill distribution and the contribution of these workers to total output is relatively small. Hence, for high minimum wages, a trade-off between higher output and higher employment emerges in the short- and medium-run.

Importantly, note that the difference between the short- and long-run effects gets amplified with higher minimum wages and is quantitatively important only beyond a Kaitz index of about 60%. This is because the number of jobs that are destroyed on impact increases disproportionately in the minimum wage. The initial spike in the unemployment rate largely determines the time needed to transition to the new stationary equilibrium. The model's predictions are thus in line with the lack of a pronounced transition path for previously observed minimum wages.[43]

In order to formalize how different planning horizons affect the assessment of higher minimum wages, Figure 13 shows how the average discounted unemployment rate (panel A) and the net present value of output (panel B) evolve as a function of the minimum wage and for different time horizons T. The lighter the line, the longer the time horizon T. The darkest line corresponds to $t = 0$ and the red dashed line corresponds to $T = \infty$. The lines in between show the change in the net present value of output and the average discounted unemployment rate taking into account only 1, 2, 5 and 10 years after the introduction of the minimum wage.

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[43]Cengiz et al. (2019) conduct event-study analyses and find no evidence for pronounced transitional dynamics in total employment over a five year window following past minimum wage hikes in the US.

FIGURE 13: Discounted Output and Employment Effects
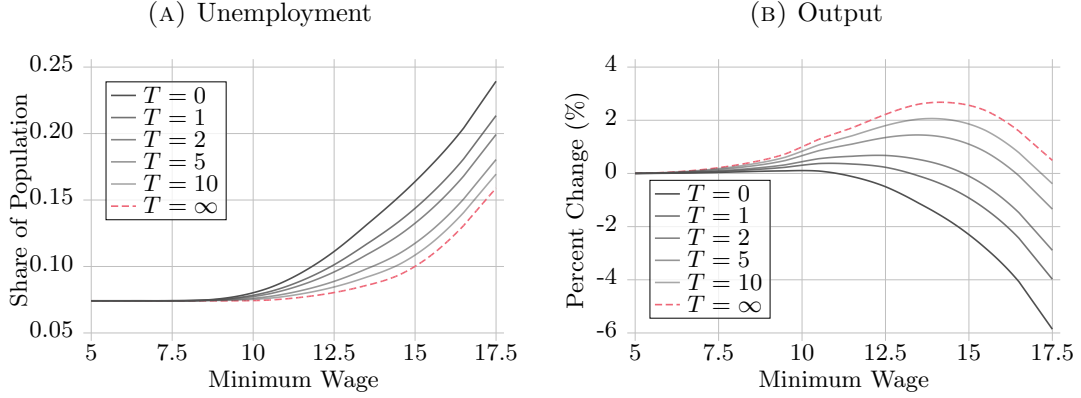
(A) Unemployment

(B) Output

Notes: This figure shows and the average discounted unemployment rate (panel A) and the net present value of output (panel B) of different minimum wage policies for different time horizons. Lighter lines correspond to longer time horizons. The red dashed line corresponds to an infinite time horizon. The average discounted unemployment rate between $t = 0$ and $t = T$ is defined as $\left(\sum_{t=0}^{T} \beta^t \right)^{-1} \sum_{t=0}^{T} \beta^t u_t(\bar{w})$ where $u_t(\bar{w})$ is the unemployment rate t periods after the minimum wage is raised from zero to $\bar{w}$. The relative change in the net present value of output is defined as $NPV(\bar{w}) = \sum_{t=0}^{T} \beta^t Y_t(\bar{w})$ and panel B plots the relative difference between $NPV(\bar{w})$ and $NPV(0)$.

Panel A of Figure 13 shows that there is no binding minimum wage that decreases the average discounted unemployment rate – even for an infinite time horizon. The small long-run reduction in the unemployment rate does not outweigh higher short-run unemployment rates. The long-run net present value of output is maximized at a minimum wage of €14.2 and 2.67 percent above the net present value of output without a minimum wage. Adopting a five- and two-year horizon, the net-present-value maximizing minimum wage drops to €13.5 and €12.3 respectively with smaller but still significant discounted output gains of 1.45 and 0.68 percent.

Starting from a Moderate Minimum Wage. Thus far, I use the equilibrium without a minimum wage as the baseline. A natural question to ask is whether the stark difference between long- and short-run effects of high minimum wages disappears when start from a typical wage floor. After all, many countries – including Germany today – already have a binding minimum wage in place. Therefore, I now assume that the economy has already converged to the stationary equilibrium with a minimum wage of €8.5 – the initial level set in 2015. With a corresponding Kaitz index of 47%, this is right in the middle of the range of current minimum wage levels in developed countries (Dube, 2019).

Figure 14 shows the response of the unemployment rate and output to hypothetical reforms that increase the minimum wage to €11.0, €13.0 and €14.4. The solid lines correspond to the case where there is no minimum wage before the reform and the dashed lines depict the case where the baseline is the stationary equilibrium with a minimum wage of €8.5. Panel A shows that starting from a typical minimum wage level does not change the unemployment response significantly. There is still a substantial spike right on impact and a lengthy adjustment process towards the new stationary equilibrium. Panel B shows that the same holds for the response of output. The intertemporal trade-off is mainly determined by the new minimum wage and is only weakly attenuated when starting from a moderate minimum wage level.

FIGURE 14: Transition Path when Starting at a Minimum Wage of 8.5 Euro
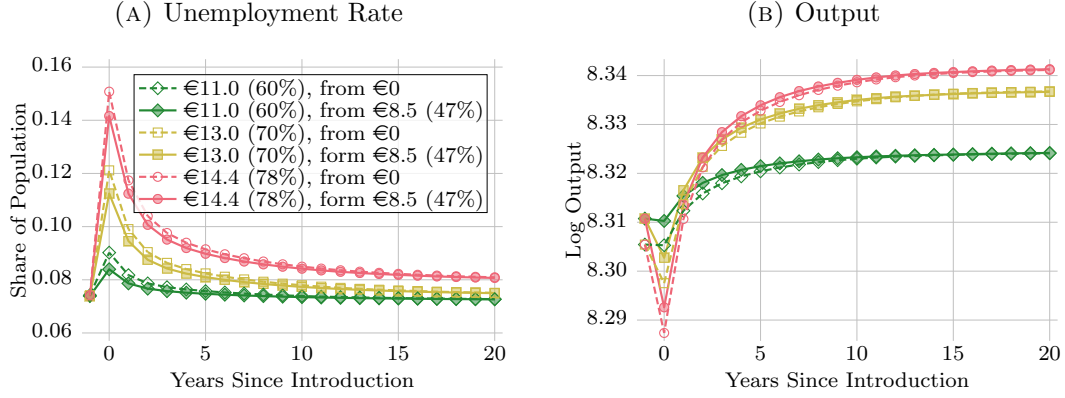
(A) Unemployment Rate   (B) Output

Notes: This figure shows how the unemployment rate and output evolve following minimum wage hikes of different magnitude. The solid lines correspond to the case where there is no minimum wage before the reform and the dashed lines depict the case where the baseline is the stationary equilibrium with a minimum wage of €8.5.

Gradual Implementation. The disconnect between short- and long-run effects naturally leads to the question as to whether gradually increasing the minimum wage can attenuate the transitional dynamics. In particular, how gradual does the implementation have to be tin order to keep the unemployment rate at a managable level?

To study this question, I focus on the transition between the baseline equilibrium wihtout a minimum wage and the €13 minimum wage equilibrium as both steady states feature the same unemployment rate. I study the transitional dynamics of a gradual: Rather than suddenly increasing the minimum wage from €0 to €13, I assume that the minimum wage increases from €0 to €8.5 on impact and then further increased to €13 over the next $T \in \{4, 12, 20, 40\}$ quarters: For $t < 0$, the minimum wage is zero, between $t = 0$ and $t = T$, the minimum wage increases such that $\bar{w} = 8.5 + t \times (13 - 8.5)/T$, and after the phase-in period $(t > T)$, the minimum wage is constant at €13. This is motivated by the finding that the transition towards a moderate minimum wage (€8.5, 47%) is negligible, but the transition from a moderate minimum wage to a high minimum wage (€13, 70%) is costly. More generally, this reflects the non-linear relationship between the share of unprofitable jobs and the minimum wage shown in Figure 11.[44]

Figure 15 shows the resulting changes in the unemployment rate and output over time for these different reforms. While the unemployment rate increases by over 4 percentage points following a sudden minimum wage hike, the maximum spike is almost halved with a 1-year gradual implementation $(T = 4)$ and falls to 0.7 percentage points with a phase-in of 5 years $(T = 20)$. With a phase-in period of 10 years, the trade-off between (transitional) unemployment and output almost disappears. Panel B shows that even a phase-in period of one year eliminates the initial recession. In fact, the output response is not vastly different for $T \in \{4, 12, 20\}$ even though transitional unemployment is significantly reduced when using a 5-year instead of a 1-year phase-in period. Going from a 5-year to a 10-year period, however, delays reallocation substantially.
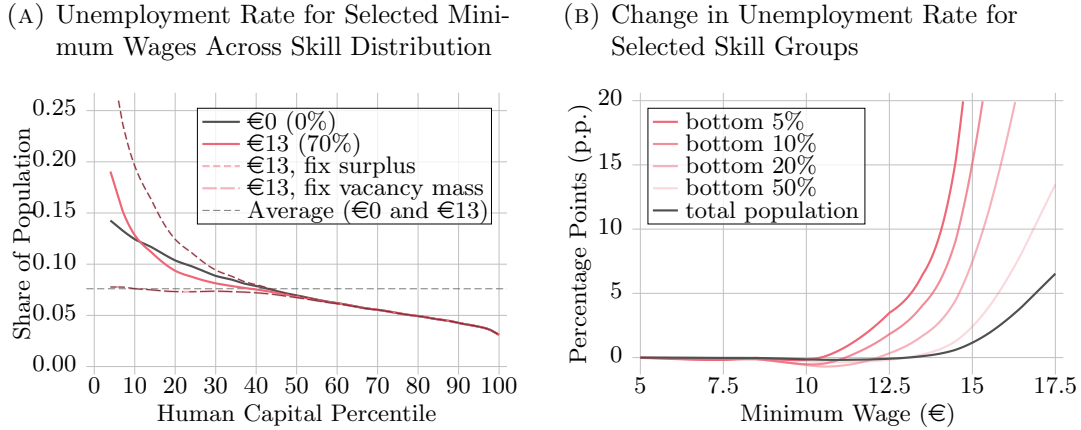
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[44] A more detailed analysis would allow for a concave path for the minimum wage where the speed of minimum wage growth slows down as more and more jobs are affected and become unprofitable.

FIGURE 15: Transition Path with Gradual Increase to €13 (70%)

(A) Unemployment Rate
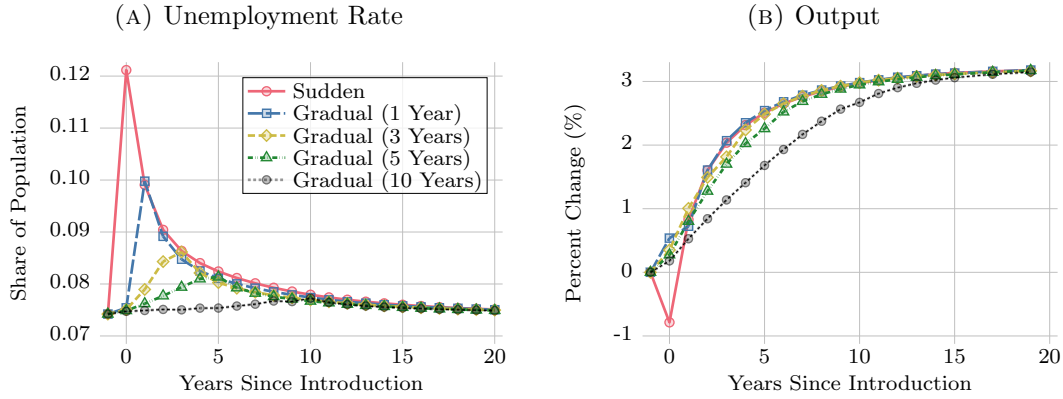
(B) Output

Notes: This figure shows how the unemployment rate and log output evolve following increases in the minimum wage from €0 to €13 implemented over different time horizons. The solid red line represents a sudden hike. The remaining lines represent a sudden hike from €0 to €8.5 followed by a linear increase in the minimum wage from €8.5 to €13 of the next $T \in \{1, 3, 5, 10\}$ years.

FIGURE 16: Employment Effects Across the Skill Distibution

(A) Unemployment Rate for Selected Minimum Wages Across Skill Distribution

(B) Change in Unemployment Rate for Selected Skill Groups

Notes: Panel A displays the unemployment rate across the skill distribution of workers in the baseline equilibrium without a minimum wage (black line), the equilibrium with a €13 minimum wage (red line). The red short-dashed line displays the partial equilibrium unemployment rate for a minimum wage of €13 when workers' surplus of employment is held fixed at the baseline level. The long-dashed red line corresponds to the case where the mass of vacancies is held fixed. Panel B shows the unemployment rate of population and of different sub-populations (in terms of their human capital) as a function of the minimum wage.

5.3 Distributional Effects of High Minimum Wages

As a final step, I analyze the distributional effects of increasing the minimum wage. The model allows me to analyze heterogeneous effects not only by workers' skill level but also by demographic characteristics.

Panel A of Figure 16 shows the unemployment rate across the worker skill distribution in the baseline equilibrium without a minimum wage and in the equilibrium with a €13 minimum wage. Recall that the overall unemployment rate is the same in these two steady states. In the baseline equilibrium, the unemployment rate decreases with human capital. This is a consequence of the skill-independent subsistence benefit level which makes the surplus of employment and hence search effort during unemployment an increasing function in the workers' skill level. With a minimum wage of €13, this relationship becomes more pronounced at the very bottom, but less

pronounced in the middle of the skill distribution. This highlights the presence of two oppsing forces. In the bottom skill segments, firms' reduction in vacancy posting outweighs the increase in workers' surplus of employment. In the middle part of the distribution, the reverse is true. The light-red dashed lines show the unemployment skill relationship when each of these forces is switched off. Without firms' vacancy reduction, the relationship becomes flatter and without the positive effect on the surplus of employment, the relationship becomes steeper.

Panel B of Figure 16 shows how the minimum wage affects the unemployment rate for different skill groups. While overall (long-run) unemployment rises above the baseline level only for minimum wage above €13, the model predicts higher unemployment for minimum wages above €10.5 (58%) among workers in the bottom 5% of the skill distribution.

Panel A of Figure 17 shows that average lifetime utility in the population increases up until a minimum wage of over €16. However, at the per capita optimum, a significant share of low-skill workers experiences large welfare losses. For the bottom 5, 10 or 20 percent of the human capital distribution, average lifetime utility peaks between €13 and €14 and declines sharply to the right of the optimum. This is because high minimum wages make low-skill workers unemployable and forces them into long-term unemployment. While low-skill workers are the ones who benefit the most from increasing the minimum wage, they also suffer the most if the minimum wage is set so high that they become unemployable.

Panel B shows how the lifetime utility changes for different demographic groups. As with the initial minimum wage, welfare gains are not distributed equally. While average lifetime utility of men and single women without kids grows strongly with higher minimum wage levels, single women with kids and married women do not benefit from the reallocation effects in terms of their lifetime utility. The latter actually experience small welfare losses for minimum wages between €10 and €14. Reallocation away from low-hours jobs comes at a disutility cost of longer working hours. This disutility is estimated to be substantially larger for single women with kids and married women reflecting the large share of non-full-time jobs among these workers in the baseline equilibrium. Time constraints due to childcare obligations that feature into this disutility thus interact with the minimum wage.

FIGURE 17: Heterogeneous Welfare Effects

(A) By Skill Group

(B) By Sociodemographics
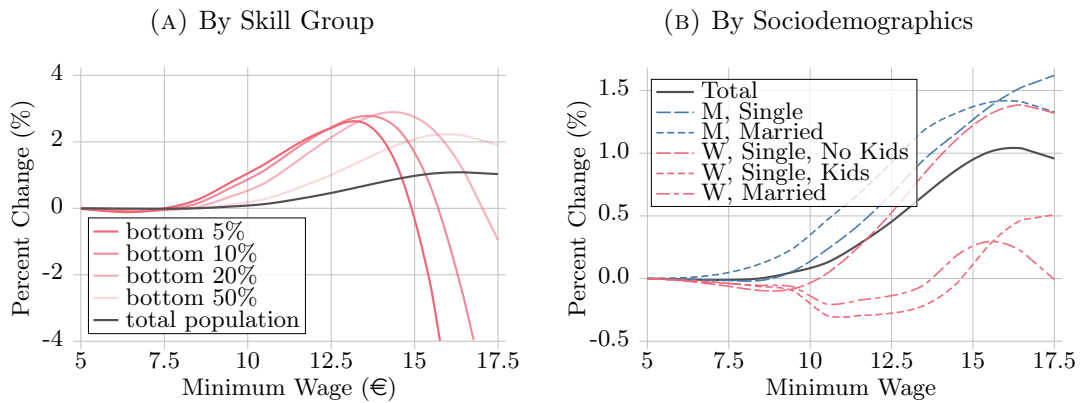
Notes: This figure shows how the minimum wage changes average lifetime utility of different sub-populations. Panel A distinguishes between different parts of the human capital distribution and panel B presents the effects by demographic characteristics.

6 Conclusion

This paper is motivated by recent proposals to increase minimum wages in developed countries and the lack of quantitative structural models that can inform policy makers.

I show that a rich model with two-sided heterogeneity, endogenous search and vacancy posting, a realistic tax- and transfer system and multiple employment levels is not only able to match important aspects of pre-reform micro data but can also replicate the available reduced form evidence on the German minimum wage introduction in 2015 (Dustmann et al., 2022).

I use the estimated and tested model to analyze how increasing the minimum wage affects employment and output in the short- and long-run. I find that the minimum wages discussed around the world can significantly improve the composition of jobs without reducing employment. Importantly, however, I also show that counterfactually high minimum wages (above a Kaitz index of 55-60%) need to be implemented gradually in order to avoid transitional unemployment. This is because, in the presence of search frictions, reallocation takes time and the share of unprofitable jobs increases non-linearly in the minimum wage.

Against the backdrop of these results, a number of avenues for future research seem particularly fruitful given the potential benefits of high minimum wages. For example, while wages in the tradable sector are relatively high in the absence of a minimum wage, high minimum wages raise the question at what point firms in the tradable sector decide to relocate to other countries. Just like for taxation, international cooperation may become important for minimum wage laws.

In addition, complementarities between low- and high-skill tasks can reduce demand for high-skill jobs as low-skill jobs become non-profitable, but may also limit the decline in vacancy posting for low-skill jobs as they are required for more important high-skill tasks. In addition, it will be fruitful to investigate how firms' investment decisions are affected by the minimum wage. Will high minimum wages lead firms to replace labor with capital and will workers find jobs at other, more productive firms? A closely related question is how high minimum wages affect endogenous human capital accumulation. On the one hand, higher minimum wages may decrease workers' incentives to invest in their education as wage differentials are reduced. On the other hand, the disappearance of jobs in low-skill segments of the labor market will increase workers' human capital accumulation.

Finally, the effects of the minimum wage interact with other labor market policies such as the design of unemployment insurance or earned income tax credits. As both unemployment benefits and minimum wages affect workers' surplus of employment, the optimal generosity of the social safety net and the level of the minimum wage should be determined jointly. The findings in this paper show that this point is particularly important in frictional labor markets where progressive tax-and-transfer schemes subsidize low-productivity jobs.

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A   Additional Tables and Figures

TABLE A.1: Sociodemographic Types

	$P_j = \Pr(j)$	$\Pr\left(g(j)\right)$	$\Pr\left(j\|g(j)\right)$
Sociodemographics			
Men, Single	0.214	0.514	0.416
Men, Married	0.300	0.514	0.584
Women, Single, No Kids	0.168	0.486	0.346
Women, Single, Kids	0.046	0.486	0.095
Women, Married	0.272	0.486	0.560

Note: The share of each sociodemographic group conditional on gender $g(j)$ is computed from the SOEP and then multiplied by the respective gender share in the SIAB data. Source: SOEP, SIAB, own calculations.

TABLE A.2: Model Fit – Job-to-Job Transitions

	full-time	part-time	mini-job
Job-to-job transition			
Data	0.028	0.034	0.088
Model	0.039	0.046	0.062
Godfather shock			
Data	0.017	0.022	0.065
Model	0.017	0.022	0.050

Note: This table shows the probability of job-to-job transitions for full-time, part-time and mini-job workers. The top panel shows the probability of any job-to-job transition and the bottom panel shows the probability of being hit by the Godfather shock. Data: SIAB.

FIGURE A.1: Fit of Estimated Tax Functions

(A) Married Men

(B) Married Women

(C) Single Women with Kids

(D) Other Singles
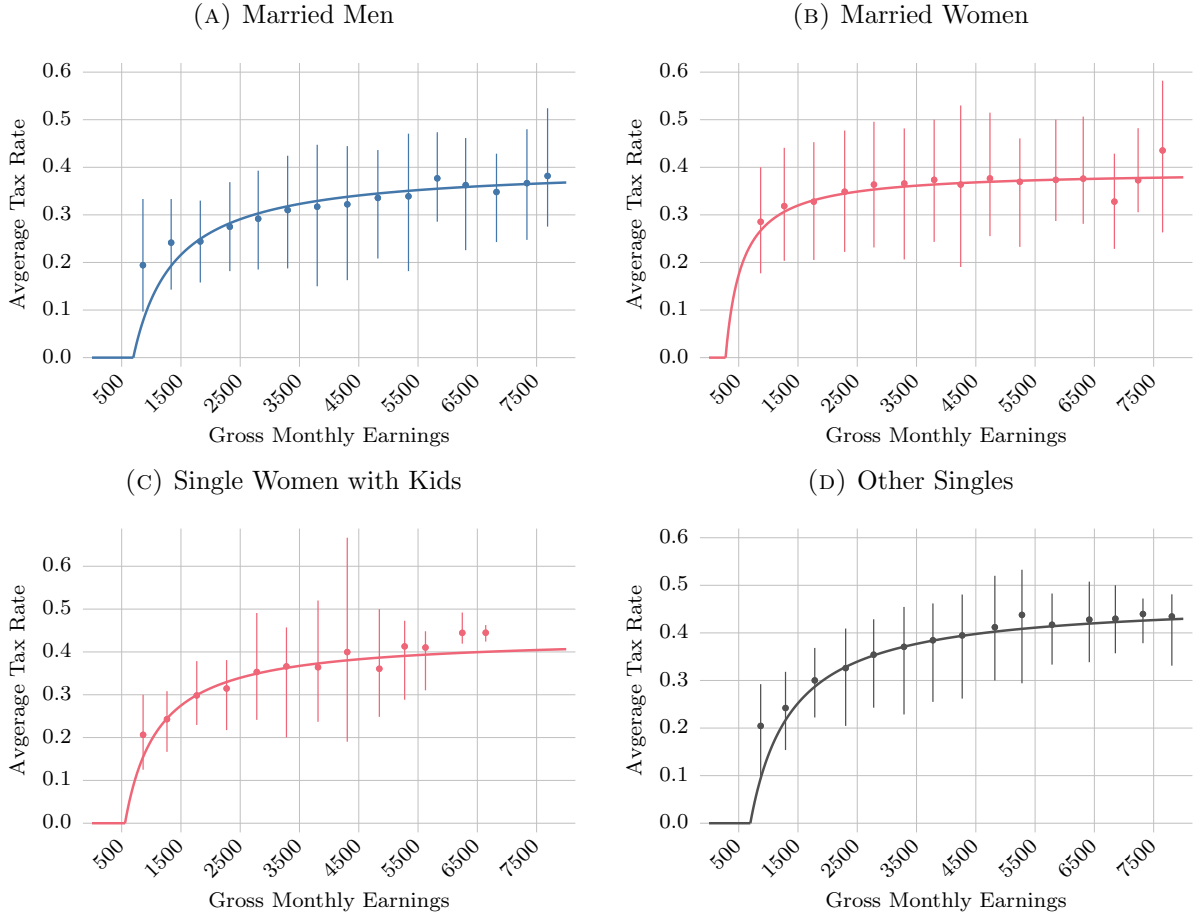
Notes: This figure shows estimated average tax functions as well as the mean average tax rate in various gross earnings bins. The spikes show the range between the 10th and 90th percentile of average tax rates in those bins. The average tax function is $T(y) = (1 - \tau^j) \max\{0, y - D^j\}/y$.

FIGURE A.2: Estimated Human Capital and Firm Productivity Distributions

(A) Workers

(B) Firms
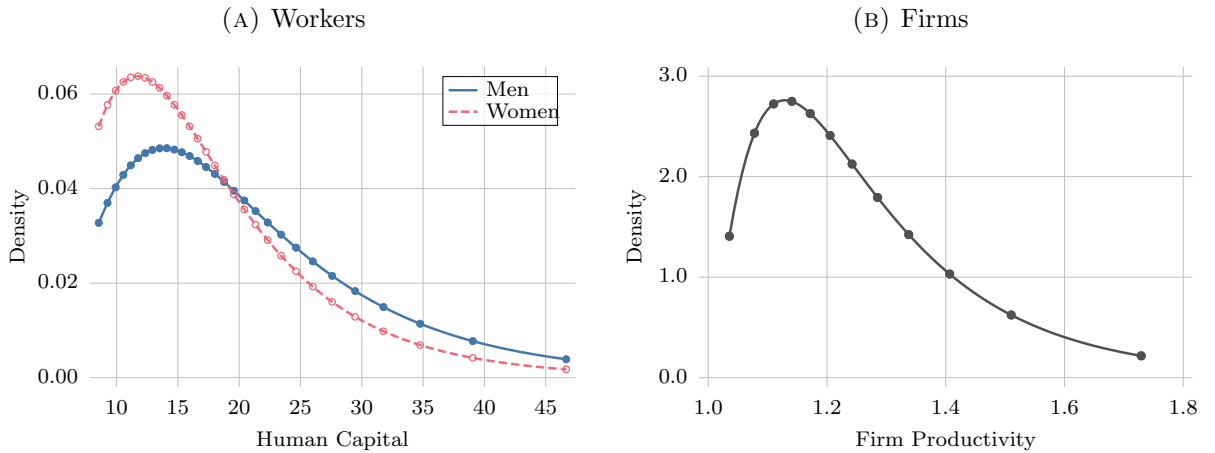
Note: This figure shows the density of the estimated human capital distributions of workers (by gender) and firm productivity distribution. All distributions are truncated log normal distributions. The markers refer to the grid points used to discretize the distribution.

TABLE A.3: Model Fit – Other Moments

	Model	Data
Job Vacancy Rate		
Job Vacancy Rate	0.034	0.025
Job Vacancy Rate (full-time)	0.029	–
Job Vacancy Rate (part-time)	0.039	–
Job Vacancy Rate (mini-job)	0.054	–
Firm Size Distribution		
Mean of log firm size	3.742	4.136
Std. dev. of log firm size	0.927	2.187
Mean of log firm size (full-time)	3.554	4.147
Std. dev. of log firm size (full-time)	1.037	2.173
Mean of log firm size (part-time)	2.171	2.976
Std. dev. of log firm size (part-time)	0.749	2.039
Mean of log firm size (mini-job)	1.144	1.927
Std. dev. of log firm size (mini-job)	0.130	1.707

Note: This table shows the job vacancy rate, and the mean and standard deviation of the log firm size distribution of different job types in the model and the estimated data. Data: BHP, own calculations.

TABLE A.4: Model Fit – Employment Status

	$\Pr(e_{ft}|e)$	$\Pr(e_{pt}|e)$	$\Pr(e_{mj}|e)$	$\Pr(u)$	$\Pr(lu|u)$
Men, Single					
Data	0.832	0.100	0.068	0.109	0.526
Model	0.851	0.085	0.064	0.079	0.553
Men, Married					
Data	0.908	0.059	0.033	0.040	0.454
Model	0.901	0.077	0.022	0.053	0.480
Women, Single, No Kids					
Data	0.666	0.224	0.110	0.068	0.520
Model	0.706	0.183	0.111	0.080	0.525
Women, Single, Kids					
Data	0.330	0.534	0.136	0.140	0.552
Model	0.236	0.620	0.144	0.105	0.529
Women, Married					
Data	0.309	0.516	0.176	0.040	0.554
Model	0.199	0.625	0.176	0.086	0.488
Total					
Data	0.663	0.240	0.096	0.064	0.518
Model	0.639	0.268	0.093	0.074	0.511

Note: This table shows the share of full-time, part-time and marginal jobs conditional on employment (columns 2-4), the unemployment rate (column 5) and the share of long-term unemployment conditional on unemployment (column 6) for each sociodemographic worker type and in the population (last panel). Data: SIAB.

TABLE A.5: Model Fit – Job finding Probabilities

	$\Pr(e'|su)$	$\Pr(e'|lu)$	$\Pr(e'|e)$
Men, Single			
Data	0.286	0.062	–
Model	0.237	0.060	–
Men, Married			
Data	0.321	0.074	–
Model	0.305	0.081	–
Women, Single, No Kids			
Data	0.321	0.065	–
Model	0.255	0.068	–
Women, Single, Kids			
Data	0.303	0.082	–
Model	0.257	0.067	–
Women, Married			
Data	0.263	0.059	–
Model	0.321	0.078	–
Total			
Data	0.296	0.067	0.035
Model	0.284	0.072	0.043

Note: This table shows the probability of finding a job out of short- and long-term un-employment as well as the job-to-job transition probability for each sociodemographic worker type and in the population (last panel). Data: SIAB.

FIGURE A.3: Productivity Distribution of Vacancies

(A) Total

(B) Full-Time

(C) Part-Time

(D) Marginal
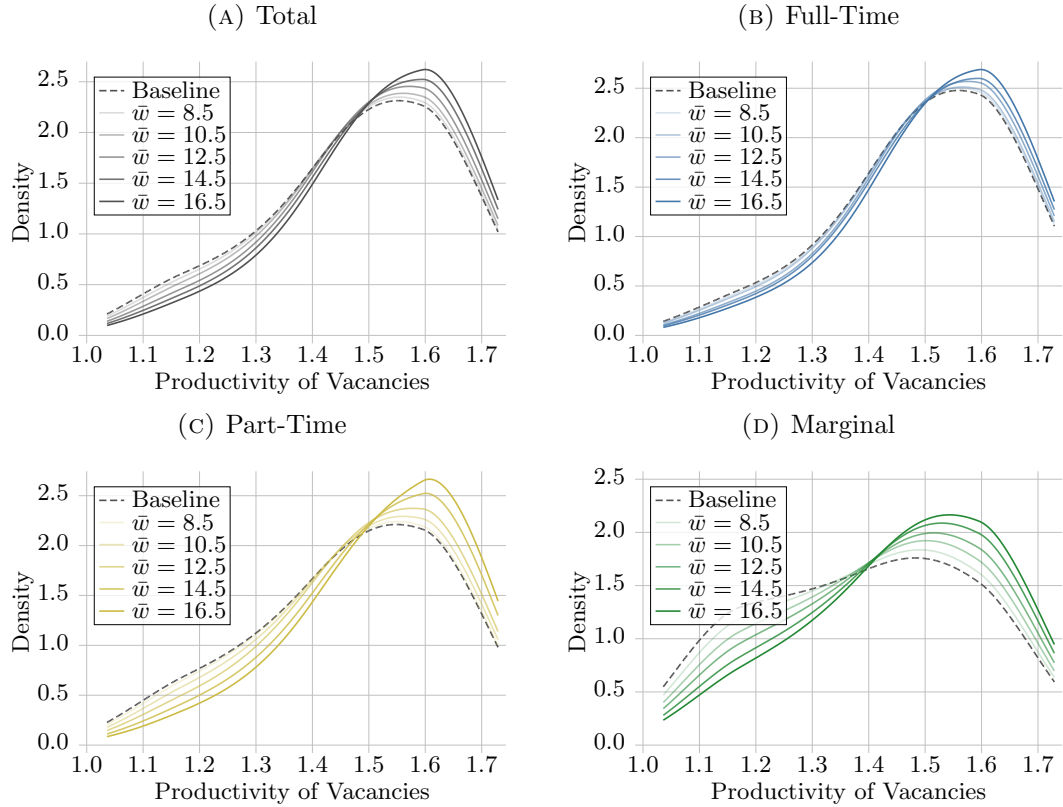
Notes: This figure shows how the productivity distribution of vacancies offered by firms changes with the minimum wage. Panel A shows the distribution for all vacancies. Panels B, C and D show the productivity distribution of full-time, part-time and mini-job vacancies respectively. I exclude skill segments in which none of the minimum wages considered is binding for any job, i.e. where all wages in the baseline equilibrium are above €16.5.

TABLE A.6: Model Fit – Job Types and Sociodemographics by Wage Groups

	[0, 5.5)		[5.5, 8.5)		[8.5, 12.5)		[12.5, 20)		[20, ∞)		Total	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Job Types												
Full-Time	0.027	0.269	0.327	0.331	0.511	0.558	0.684	0.708	0.765	0.802	0.639	0.663
Part-Time	0.000	0.239	0.267	0.257	0.333	0.289	0.272	0.262	0.230	0.191	0.268	0.240
Marginal	0.973	0.492	0.406	0.412	0.156	0.152	0.045	0.030	0.005	0.007	0.093	0.097
Gender												
Men	0.252	0.312	0.367	0.302	0.433	0.416	0.519	0.519	0.617	0.642	0.520	0.517
Women	0.748	0.688	0.633	0.698	0.567	0.584	0.481	0.481	0.383	0.358	0.480	0.483
Sociodemographics												
Men, Single	0.154	–	0.161	–	0.171	–	0.211	–	0.255	–	0.213	–
Men, Married	0.098	–	0.206	–	0.263	–	0.308	–	0.362	–	0.307	–
Women, Single, No Kids	0.230	–	0.198	–	0.180	–	0.174	–	0.143	–	0.167	–
Women, Single, Kids	0.063	–	0.061	–	0.055	–	0.043	–	0.035	–	0.044	–
Women, Married	0.456	–	0.375	–	0.332	–	0.264	–	0.204	–	0.269	–

Note: This table shows the distribution of job-types, gender and sociodemographics within different parts of the wage distribution. Data: SIAB.

TABLE A.7: Model Fit – Wage Groups by Job Types and Sociodemographics

	[0, 5.5)		[5.5, 8.5)		[8.5, 12.5)		[12.5, 20)		[20, ∞)	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Job Types										
Full-Time	0.000	0.007	0.046	0.048	0.169	0.156	0.362	0.367	0.422	0.422
Part-Time	0.000	0.018	0.090	0.103	0.263	0.224	0.343	0.376	0.303	0.279
Marginal	0.067	0.097	0.396	0.442	0.355	0.317	0.163	0.117	0.019	0.027
Gender										
Men	0.003	0.011	0.064	0.056	0.176	0.150	0.338	0.347	0.419	0.436
Women	0.010	0.025	0.119	0.140	0.250	0.227	0.339	0.346	0.282	0.262
Sociodemographics										
Men, Single	0.005	–	0.068	–	0.169	–	0.336	–	0.422	–
Men, Married	0.002	–	0.061	–	0.181	–	0.340	–	0.416	–
Women, Single, No Kids	0.009	–	0.107	–	0.228	–	0.353	–	0.303	–
Women, Single, Kids	0.009	–	0.124	–	0.260	–	0.328	–	0.279	–
Women, Married	0.011	–	0.126	–	0.262	–	0.333	–	0.269	–
Total	0.006	0.018	0.090	0.097	0.211	0.187	0.339	0.347	0.353	0.352

Note: This table shows the share of workeres in different wage groups conditional on job types, gender and sociodemographics. Data: SIAB.

TABLE A.8: Model Fit – Worker Clustered AKM Fixed Effects

	Total		Men		Women	
	Model	Data	Model	Data	Model	Data
P05 / P50	0.524	0.622	0.513	0.640	0.554	0.601
P10 / P50	0.587	0.687	0.576	0.699	0.615	0.677
P20 / P50	0.693	0.778	0.686	0.783	0.715	0.769
P30 / P50	0.792	0.854	0.787	0.854	0.806	0.850
P40 / P50	0.892	0.925	0.890	0.924	0.900	0.926
P60 / P50	1.122	1.088	1.124	1.092	1.114	1.080
P70 / P50	1.269	1.203	1.273	1.215	1.252	1.178
P80 / P50	1.460	1.370	1.464	1.393	1.435	1.314
P90 / P50	1.743	1.651	1.735	1.653	1.721	1.546
P95 / P50	1.952	1.864	1.928	1.884	1.957	1.760

Note: This table shows the median and selected percentile ratios of AKM worker fixed effects for full-time jobs. Data: SIAB.

TABLE A.9: Model Fit – Firm Clustered AKM Fixed Effects

	Full-Time		Part-Time		Marginal	
	Model	Data	Model	Data	Model	Data
P50 / P50$_{ft}$	1.000	1.000	0.932	0.993	0.844	0.851
P05 / P50	0.805	0.702	0.804	0.689	0.857	0.683
P10 / P50	0.854	0.762	0.835	0.784	0.876	0.789
P25 / P50	0.935	0.877	0.905	0.905	0.922	0.915
P75 / P50	1.076	1.084	1.085	1.125	1.105	1.231
P90 / P50	1.138	1.171	1.181	1.152	1.197	1.312
P95 / P50	1.158	1.171	1.223	1.245	1.261	1.344

Note: This table shows the median and selected percentile ratios of (full-time) firm productivity for full-time, part-time and marginal jobs. The full-time firm productivity is the exponential of the AKM firm fixed effects estimated on wages of full-time workers only. Data: SIAB.

TABLE A.10: Model Fit – Wages

|	Total		Full-Time		Part-Time		Marginal		Men		Women	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Mean	18.52	18.54	20.32	20.38	17.30	9.23	9.61	9.23	20.14	20.78	16.77	16.13
Variance (logs)	0.49	0.51	0.46	0.47	0.46	0.48	0.35	0.37	0.49	0.50	0.48	0.49
P01	5.81	5.01	6.82	5.92	6.67	5.04	4.76	4.30	6.30	5.41	5.50	4.80
P05	7.41	6.79	8.65	8.37	7.80	6.81	5.33	4.97	8.00	7.90	6.96	6.24
P10	8.57	8.18	9.91	9.92	8.56	8.11	5.80	5.51	9.28	9.60	7.98	7.35
P15	9.65	9.32	11.13	11.21	9.47	9.13	6.19	5.92	10.45	11.00	8.85	8.25
P20	10.45	10.42	12.08	12.33	10.25	10.06	6.55	6.27	11.55	12.18	9.70	9.09
P30	12.26	12.52	13.88	14.47	11.60	11.79	7.33	6.92	13.64	14.41	11.21	10.82
P40	14.29	14.63	16.12	16.42	13.30	13.52	8.00	7.55	15.65	16.44	12.75	12.69
P50	16.34	16.67	18.09	18.36	15.23	15.30	8.85	8.20	17.81	18.56	14.59	14.70
P70	21.87	21.48	23.93	23.32	20.03	19.53	10.69	9.86	23.93	23.99	19.39	19.03
P90	31.74	31.66	34.00	33.72	29.17	27.59	14.33	13.86	34.03	34.77	29.03	26.84
P95	37.22	36.00	40.71	37.27	34.19	33.28	16.56	16.86	40.90	39.19	34.02	31.88

Note: This table shows the mean wage, variance and selected percentile of hourly wages in the data and the estimated model. The variance is taken over the log wages. The moments for men and women were targeted in the estimation. Data: SIAB.

TABLE A.11: Minimum Wage Effects – Mechanisms

|	(1) Baseline	(2) GE	(3) Fix Search	(4) Fix Vacancies	(5) Fix Surplus	(6) Fix Vac. Shares	(7) Fix Vac. Mass
	Value	Change	Change	Change	Change	Change	Change
Unemployment							
Unemployment Rate	7.42%	-0.034	0.026	-0.066	0.105	0.063	-0.174
$\Pr(e\|u)$	17.57%	-0.137	-0.239	0.162	-0.446	-0.184	0.267
$\Pr(su\|e)$	1.41%	-0.018	-0.014	-0.001	-0.015	-0.002	-0.015
Employment Level							
Log Hours	3.389	0.015	0.012	0.001	0.012	0.001	0.012
Full-Time Share	63.94%	0.435	0.251	0.031	0.296	0.107	0.416
Part-Time Share	26.79%	0.803	0.789	0.008	0.776	-0.011	0.560
Marginal Share	9.27%	-1.238	-1.040	-0.040	-1.073	-0.096	-0.976
Wages, Earnings & Incomes							
Mean Log Wages	2.797	0.020	0.021	0.017	0.021	0.018	0.019
Mean Log Productivity (p)	0.376	0.004	0.007	-0.001	0.007	-0.001	0.005
Mean Log Productivity ($\alpha_x p$)	0.382	0.006	0.008	-0.001	0.008	-0.001	0.006
Mean Log Earnings	7.652	0.035	0.032	0.017	0.033	0.019	0.031
Mean Log Net Earnings	7.297	0.028	0.027	0.014	0.027	0.016	0.025
Mean Log Income	7.598	0.008	0.007	0.003	0.008	0.004	0.007
Macro Aggregates							
Log Output	8.305	0.005	0.005	-0.000	0.005	-0.001	0.006
Log Transfers	4.549	-0.059	-0.048	-0.031	-0.043	-0.023	-0.065
Log Labor Taxes	6.750	0.009	0.008	0.004	0.008	0.004	0.009

Note: This table shows the long-run effect of the introduction of a federal minimum wage of 8.5 EUR relative to the baseline equilibrium without a minimum wage (first column). The second columns shows the new general equilibrium. In the remaining columns, different margins of adjustment are switched off one at a time. In column 3, workers' search effort is held fixed. In column 4, firms' vacancy posting is held fixed. In column 5, workers can adjust their search effort, but the expected surplus of meeting a firm is held constant (search effort will still react to changes in labor market tightness). In column 6, the hours-and productivity shares of posted vacancies are held fixed, but the total mass of vacancies is allowed to adjust. In column 7, the total mass of vacancies is held fixed, but the hours- and productivity distribution of vacancies is allowed to adjust. In all scenarios, unprofitable vacancies may not be posted. Changes refer to the absolute difference to the baseline outcome (e.g. percentage points or log points).

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FIGURE A.4: Employment and Output Effects for Alternative Values of the Vacancy-Elasticity of the Matching Function
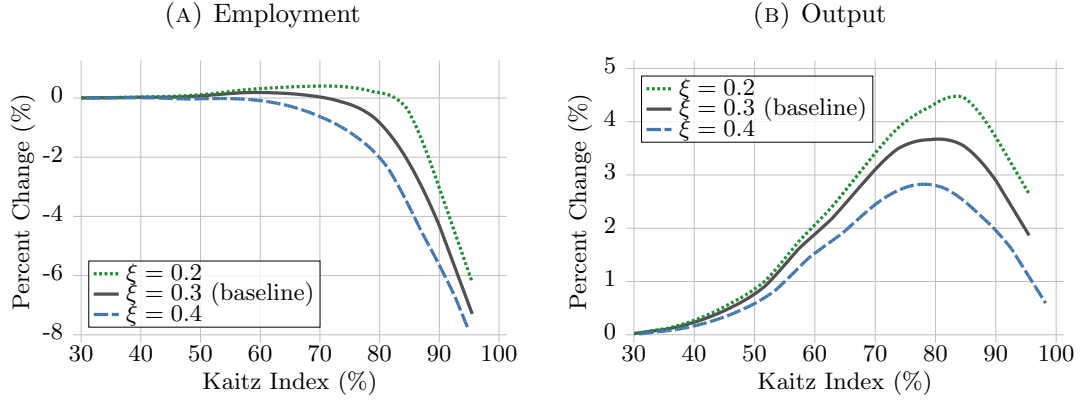
(A) Employment

(B) Output

Notes: This figure shows the predicted relative changes in total employment and output as a function of the minimum wage's Kaitz index for alternative values of the elasticity of the matching function with respect to labor market tightness, ξ. Starting from the estimated model, I only change ξ from its baseline value of 0.3 (black, solid line) to 0.2 (green, short-dashed line) and 0.4 (blue, long-dashed line). Since this changes the wage distribution in the equilibrium without a minimum wage, I report the Kaitz index on the horizontal axis.

FIGURE A.5: Employment and Output Effects for Alternative Values of the Wage Piece Rate
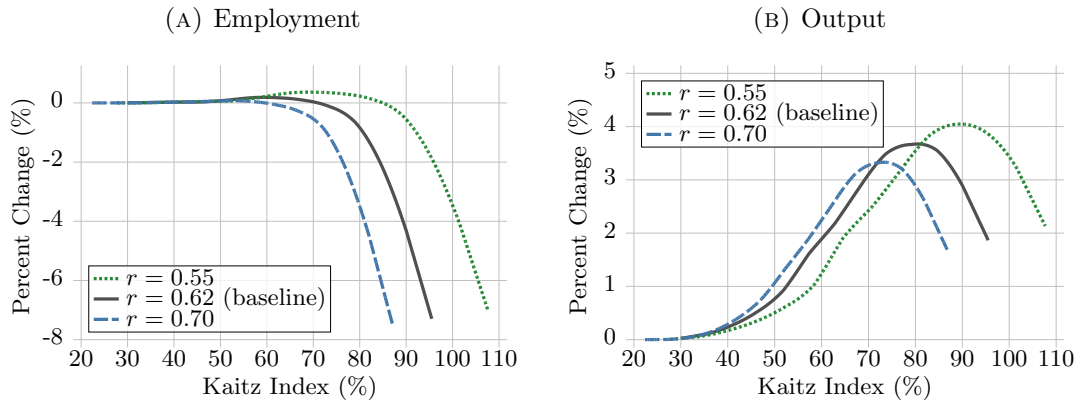
(A) Employment

(B) Output

Notes: This figure shows the predicted relative changes in total employment and output as a function of the minimum wage's Kaitz index for alternative values of the piece rate, r for full-time and part-time jobs. Starting from the estimated model, I only change r from its baseline value of 0.62 (black, solid line) to 0.55 (green, short-dashed line) and 0.7 (blue, long-dashed line). The piece rate of marginal jobs is adjusted using the same adjustment factor implied by the change for full-time and part-time piece rates. Since this changes the wage distribution in the equilibrium without a minimum wage, I report the Kaitz index on the horizontal axis.

Online Appendix

B   Details of the Estimation Procedure

Free Parameters   The set of parameters that need to be jointly estimated includes the following 40 parameters:

- vacancy creation cost parameters κ_1 and κ_{2x} for $x = f, p, m$
- mass of firms m_{firm}
- mean μ_p and standard deviation σ_p of the distribution of log firm productivity
- search efficiency parameters ϕ_e, ϕ_{su} and ϕ_{lu}
- gender-specific mean μ_h^g and σ_h^g of the distribution of log human capital
- state (dis-)utility parameters γ_f^j, γ_p^j and γ_m^j for each sociodemographic group
- search disutility parameter ζ_2
- CRRA parameter γ_c

Targeted Moments   The set of moments targeted in the estimation includes the following (sets of) moments grouped into four subsets:

- overall worker moments

    - average UI-elasticity of the job finding probability of short-term unemployed workers
    - unemployment rate
    - share of long-term unemployed conditional on unemployment
    - share of part-time jobs conditional on employment
    - share of mini-jobs conditional on employment
    - average job finding probability of short-term unemployed workers
    - average job finding probability of long-term unemployed workers

- worker moments by sociodemographic group

    - unemployment rate
    - share of long-term unemployed conditional on unemployment
    - share of part-time jobs conditional on employment
    - share of mini-jobs conditional on employment
    - average job finding probability of short-term unemployed workers
    - average job finding probability of long-term unemployed workers

- firm moments

    - mean and standard deviation of log firm size
    - mean and standard deviation of log full-time firm size

- distributional moments

    - part-time and mini-job share in each wage quintile

- $0.05, 0.1, \ldots, 0.9, 0.95$ quantiles of the gender specific wage distribution
- ratio of selected quantiles $(0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.75, 0.9, 0.95, 0.99)$ to the median of the gender specific worker fixed effect distribution and the firm fixed effect distribution of full-time workers
- variance of log full-time wages (net of the residual variance), variance of the worker fixed effects, variance of the firm fixed effects and the correlation between the worker and firm fixed effects
- shares of the the worker, firm and covariance component of the variance of log full-time wages

Objective Function Let θ be an arbitrary vector of (free) parameters and $m_k(\theta)$ the kth model moment in a set (e.g. firm moments) when the model is solved with free parameters p. We compute the relative deviation of the model moment from its data target d_k as

$$f_k(\theta) = \frac{m_k(\theta) - d_k}{\frac{1}{2}\big(|m_k(\theta)| + |d_k|\big) + \psi_k}$$

where $\psi_k \geq 0$ is an adjustment factor (e.g. 0.05) to avoid numerical problems when the moment is close to zero. For each set of moments we compute the weighted sum of squared percent deviations between model and data as

$$F_{set}(\theta) = \sum_{k=1}^{K_{set}} w_k f_k(\theta)^2$$

where w_k is the weight of the kth moment in the set and all weights sum to one within each set. The overall objective value is given by

$$F(\theta) = \sum_{set} w_{set} F_{set}(\theta)$$

$$= 0.075 F_{workers}(\theta) + \frac{0.45}{J} \sum_{j=1}^{J} F_{workers_j}(\theta) + 0.025 F_{firm}(\theta) + 0.45 F_{dist}(\theta)$$

C Data and Target Moments

C.1 Targeted Moments

The set of moments targeted in the estimation includes the following (sets of) moments grouped into four subsets:

- overall worker moments (weight: 0.075)

  - average UI-elasticity of the job finding probability of short-term unemployed workers (2/14)
  - unemployment rate (2/14)
  - share of long-term unemployed conditional on unemployment (2/14)
  - share of part-time jobs conditional on employment (1/14)
  - share of marginal employment conditional on employment (1/14)
  - average job finding probability of short-term unemployed workers (1/14)
  - average job finding probability of long-term unemployed workers (1/14)
  - average job finding probability of full-time, part-time and marginally employed workers (2/14 each)

- worker moments by sociodemographic group (weight: 0.45)

  - unemployment rate (1/6)
  - share of long-term unemployed conditional on unemployment (1/6)
  - share of part-time jobs conditional on employment (1/6)
  - share of mini-jobs conditional on employment (1/6)
  - average job finding probability of short-term unemployed workers (1/6)
  - average job finding probability of long-term unemployed workers (1/6)

- firm moments: (weight: 0.025)

  - mean and standard deviation of log firm size
  - mean and standard deviation of log full-time firm size

- distributional moments: (weight: 0.45)

  - part-time and mini-job share in each wage quintile
  - $0.05, 0.1, \ldots, 0.9, 0.95$ quantiles of the gender specific wage distribution
  - ratio of selected quantiles $(0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.75, 0.9, 0.95, 0.99)$ to the median of the gender specific worker fixed effect distribution and the firm fixed effect distribution of full-time workers
  - variance of log full-time wages (net of the residual variance), variance of the worker fixed effects, variance of the firm fixed effects and the correlation between the worker and firm fixed effects
  - shares of the the worker, firm and covariance component of the variance of log full-time wages

C.2 Objective Function

Let θ be an arbitrary vector of (free) parameters and $m_k(\theta)$ the kth model moment in a set (e.g. firm moments) when the model is solved with free parameters p. We compute the relative deviation of the model moment from its data target d_k as

$$f_k(\theta) = \frac{m_k(\theta) - d_k}{\frac{1}{2}\big(|m_k(\theta)| + |d_k|\big) + \psi_k}$$

where $\psi_k \geq 0$ is an adjustment factor (e.g. 0.05) to avoid numerical problems when the moment is close to zero. For each set of moments we compute the weighted sum of squared percent deviations between model and data as

$$F_{set}(\theta) = \sum_{k=1}^{K_{set}} w_k f_k(\theta)^2$$

where w_k is the weight of the kth moment in the set and all weights sum to one within each set. The overall objective value is given by

$$F(\theta) = \sum_{set} w_{set} F_{set}(\theta)$$

$$= 0.075 F_{workers}(\theta) + \frac{0.45}{J} \sum_{j=1}^{J} F_{workers_j}(\theta) + 0.025 F_{firm}(\theta) + 0.45 F_{dist}(\theta)$$

C.3 Data

SIAB. I mainly rely on high-quality administrative data from the IAB (Institute for Employment Research of the German Federal Employment Agency). In particular, I use the SIAB (1975-2014) data which is a two percent random sample of the integrated employment biographies collected at the IAB.[45] I use the years 2011 to 2014. The data covers all individuals in Germany, which are employed, receive unemployment benefits, are officially registered as job-seeking at the German Federal Employment Agency or (plan to) participate in programs of active labor market policies. The only workers not included in the IAB data are civil servants as they are not subject to social security contributions. Marginally employed workers, however, are included in the data even though they are also not subject to social security contributions. Information on earnings is top-coded at the social security contribution limit. This affects about 10% of all workers each year. Following Card et al. (2013), I impute top-coded earnings using Tobit regressions by year, gender, east/west, age groups and education groups.

SOEP. I complement the SIAB data with survey data from the German Socio-Economic Panel (SOEP v34, 1984-2017). The (SOEP) study is a representative longitudinal study of private households administered by the German Institute for Economic Research (DIW). The

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[45]See Antoni et al. (2016) for a detailed description of the data. The data are confidential and can only be accessed after signing a confidentiality agreement (https://fdz.iab.de/en/data-access/).

data contains annual information on more than 15 thousand workers. In the SOEP, I drop civil servants in order to be consistent with the IAB data.[46].

BHP. I also use data from the Establishment History Panel (1975-2014) provided via the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB). These data are used to compute summary statistics for the distribution of log firm size.[47]

Job Vacancy Survey. To compute the job vacancy rate (number of vacancies divided by the sum of vacancies and jobs), I use aggregate and publicly available data provided by the IAB and based on their job vacancy survey for the year 2014.[48] I multiply the number of reported vacancies by the inverse of the reporting frequency (0.5) which gives a job vacancy rate of 0.025.

C.4 Computation of Targeted Moments in the Data

Sociodemographics The distribution of sociodemographic types conditional on gender $\Pr(j|g)$ is taken from the SOEP. The distribution of gender $\Pr(g)$ is taken from the SIAB in order to use as much administrative information as possible.

Labor Market States As the SIAB data does not contain sociodemographic information for employed workers, I have to fill some gaps with information from the SOEP while ensuring that the joint distribution of gender and labor market status remains consistent with the administrative SIAB data.

I start by computing the unemployment rate conditional on j such that it is consistent with the gender-specific unemployment rate in the SIAB

$$\Pr(u|j) = \Pr(u|j,g) = \frac{\Pr(u,j|g)}{\Pr(j|g)} = \frac{\Pr(j|u,g)\Pr(u|g)}{\Pr(j|g)} \tag{C.1}$$

where only $\Pr(j|g)$ is taken from the SOEP. The probability of long-term unemployment conditional, $\Pr(lu|u,j)$, is taken directly from the SIAB.

Computing the share of type-j workers who have a type-x job requires slightly more information from the SOEP:

$$\Pr(e_x|j) = \Pr(e_x|j,g) = \frac{\Pr(j|e_x,g)\Pr(e_x|g)}{\Pr(j|g)} \tag{C.2}$$

Here, only $\Pr(j|e_x,g)$ and $\Pr(j|g)$ are taken from the SOEP.

Transition Probabilities The job finding rate out of short- and long-term unemployment, $\Pr(e'|su,j)$ and $\Pr(e'|lu,j)$, can be computed using SIAB data only. As I do not target job-to-

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job transition probabilities by sociodemographics, they are computed as the share of workers who change their employer or job type.

Hourly Wage Quantiles   To compute hourly wages based on daily earnings reported in the SIAB data, I impute average hours worked per day using data from the SOEP and job-type dependent averages reported by Dustmann et al. (2020) who have confidential information on hours for the social security data in 2014.

The average adjusted hours for full- and part-time jobs in Dustmann et al. (2020) are almost identical to the averages in the SOEP and Structure of Earnings Survey (SES).[49] The only difference between the three data sets is that, for mini-jobs, average hours worked are substantially higher in the SOEP.

For full-time jobs, I set daily hours to 7.8 which corresponds to 39 hours per week. For par-time and mini-jobs, I use the joint distribution of hours and earnings from the SOEP to take into account that some of the variation in earnings is driven by heterogeneity in hours worked. To that end, I compute the mean and standard deviation of contractual hours worked within different earnings bins. I then draw hours worked from a Normal distribution with these parameters and impose that weekly hours for part-time and mini-jobs be in the interval $[5, 35]$ and $[2, 20]$ respectively.[50] Finally I rescale the hours worked such that, on average, part-time employees work 24 hours and mini-job employees 8.7 hours per week – as reported in Dustmann et al. (2020).

Hourly wages are then computed as earnings divided by imputed hours worked. I target the 0.01, 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.7, 0.9, and 0.95 quantiles of the wage distributions conditional on job type and conditional on gender (separately). In addition, I target the share of part-time and mini-jobs and the share of men in the following five wage groups $(0, 6.5)$, $[6.5, 8.5)$, $[8.5, 12.5)$, $[12.5, 20)$, $[20, \infty)$.

Worker and Firm Fixed Effects   In the absence of a minimum wage, the wage equation in my model is very simple. As in Abowd et al. (1999) (henceforth AKM), the wage w of a full-time worker employed at firm with productivity p is log-additive in her skill h and the firm's productivity

$$\log(w) = \log(r) + \log(h) + \log(p) \tag{C.3}$$

where r is the exogenous piece-rate. I estimate the empirical distribution of worker and firm-class fixed effects using a clustered AKM approach (Bonhomme et al., 2019).

While the model is consistent with an AKM-style wage equation (Abowd et al., 1999; Card et al., 2013), I do not estimate the model by straight AKM because of two distinct reasons. First, while the SIAB data is large compared to survey data sets, it covers only 2% of all workers and the firms they are employed at. This implies that the connected set of firms and workers on which firm and worker fixed effects can be identified is too small. Second, estimation would

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[49]Dustmann et al. (2020) adjust the raw contracted working hours in the social security records to account for differences in whether sick leave and overtime are included in the contractual hours.

[50]For part-time jobs, I use 500, 750, 1000, 1500, ..., 4000, 5000, 10000 Euro as cutoffs to define the monthly earnings bins. For mini-jobs, I use the cutoffs 100, 150, ..., 500 Euro.

suffer from severe incidental parameter bias as the number of movers between two firms tends to be low.

Instead, I estimate the empirical distributions of worker and firm heterogeneity using the approach recently proposed by Bonhomme et al. (2019) (henceforth BLM) which solves both of these issues using dimension reduction techniques. The proposed method is particularly useful as it can be applied to data sets that cover only few firm-to-firm moves. The key assumption is that unobserved firm heterogeneity operates on the level of discrete firm classes rather than on the level of individual firms. Given an estimated partition of all firms into classes, firm class and worker fixed effects are identified from job-to-job transitions between firms of different classes rather than between different firms. This allows estimation of worker and firm (class) effects on much smaller samples of linked employer-employee data such as the SIAB (2%).

Class membership is estimated using K-means clustering that minimizes the within-class variation of within-firm earnings distributions:

$$\min_{k(1),\ldots,k(J),H_1,\ldots,H_K} \sum_{j=1}^{J} \frac{1}{M} \sum_{m=1}^{M} \left(F_j^m - H_{k(j)}^m \right)^2$$

where $k(j)$ is the class of firm j, F_j^m is an observable characteristic of firm j and H_k^m is the average of that characteristic across all firms in class k. I classify firms based on information on the within-firm wage distribution. In particular, I use the mean, selected percentiles (25, 50, 75) and the share of workers with top-coded earnings for full-time employees.[51] Consistent with the model where firm productivity is deterministic, I average these characteristics at the firm level over the years 2011 to 2014. This yields a time-invariant classification of firms.

Given the firm classification, I estimate the worker and firm-class fixed effects, i.e. run a clustered AKM estimation without covariates (except time fixed effects).

$$\log(w_{it}) = \alpha_i + \psi_{k(j(it))} + \gamma_t + \varepsilon_{it} \tag{C.4}$$

I then target the distribution of α conditional on gender and the worker-weighted distribution of ψ to inform the distributions of human capital and firm productivity. In particular, I target the quantile ratios $q_x^k/q_x^{0.5}$ for $k = 0.01, 0.05, 0.1, 0.3, 0.7, 0.9, 0.95, 0.99$ and $x \in \{f, p, m\}$, where q_x^k is the k-quantile of the distribution of ψ weighted by the firm's number type-x workers. In addition, I target $q_x^{0.5}/q_f^{0.5}$ for $x \in \{p, m\}$. Finally, I target the shares of the variance of log wages explained by the worker and firm components as well as the correlation between worker and firm fixed effects.

Firm Size The mean and standard deviation of log firm size are computed using administrative data from the Establishment History Panel. For consistency with the worker moments, I only consider employees between 25 and 60 years of age and drop firms that do not have employees in this age range.

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[51]This information is made available for every firm such that the within-firm earnings distribution can be approximated without observing a representative sample of employees for each firm.

Job Vacancy Rate The job vacancy rate is the number of vacancies relative to the sum of vacancies and jobs. As many vacancies are not officially registered, I do not rely on the job vacancy rate reported by Eurostat but rather use the Job Vacancy Survey (JVS).[52] The JVS contains both registered and unregistered vacancies – each account for roughly half of the total number of vacancies. In 2014, around 900 thousand vacancies were open. With roughly 36 million jobs, this gives a job vacancy rate of 2.44%.[53]

D Computational Details

D.1 Steady State

In order to compute a stationary equilibrium in the economy, I discretize the state space by using a grid of values for human capital h (30 grid points) and firm productivity p (15 grid points). I solve for the equilibrium in each skill-segment separately using the following algorithm:

1. Guess an initial distribution of vacancies across firm productivities and employment levels $(n^0(x, p))$, and a level of labor market tightness (θ^0).

2. Set $i = 0$

3. Taking as given the vacancy shares $n^i(x, p)$ and labor market tightness θ^i

   (a) Use equations (9), (10), (11) and (12) to solve for workers' search policies $\ell^i(j, \sigma)$ and value functions $V^i(j, \sigma)$ where σ is a point in the state space of a worker with skills h and demographics j (policy function iteration).

   (b) Compute the implied distribution of workers across states, $F(j, \sigma)$, using the search policies, equation 13, and the exogenous transition probabilities.

   (c) Compute the implied total search mass S, the search mass willing to accept a (x, p) job offer, $S(x, p)$ and the vacancy filling probabilities $\eta(x, p)$ from equations 5 and 16 and the probability that a (x, p)-job filled by a type-j worker is destroyed from equation 14.

   (d) Solve for firms' optimal vacancy policies $v^i(x, p)$ using equation (20).

   (e) Compute the implied vacancy shares $n^{i+1}(x, p)$ and labor market tightness θ^{i+1}.

4. If $n^{i+1}(x, p) \approx n^i(x, p)$ for all $x \in \{ft, pt, mj\}$ and for all p on the firm productivity grid and if $\theta^{i+1} \approx \theta^i$, stop! If not, set increment i repeat step 3!

In order to study the long-run effects of the minimum wage, I solve for the stationary equilibrium corresponding to minimum wages of €8.5, €9.5, ..., €17.5 and then interpolate the key statistics such as output or the unemployment rate using cubic splines.

D.2 Transition Path

Starting from the terminal stationary equilibrium, I guess a path for all equilibrium objects and solve backwards. We focus on one generic skill segment h and drop h form the notation to improve readability.

Firm Problem   Assuming that the new stationary equilibrium is reached after T periods, the firm's expected value of an employment relationship with a type-j worker starting in period t is:

$$W_t^j(x,p) = y + \sum_{s=t+1}^{T-1} \beta^{s-t} \underbrace{\prod_{k=t}^{s-1}(1 - \delta_k^j(x,p))}_{\text{Pr(survival until } s)} y(x,p) + \beta^T \underbrace{\prod_{k=t}^{T-1}(1 - \delta_k^j(x,p))}_{\text{Pr(survival until } T)} W_T^j(x,p)$$

$$\text{with} \quad W_T^j(x,p) \equiv W^{j*}(x,p) = \frac{y(x,p)}{1 - \beta(1 - \delta^{j*}(x,p))}$$

Given $W_{t+1}^j(x,p)$ and taking as given the vacancy filling rates $\eta_t^j(x,p)$, the firm optimally chooses the number of vacancies $v_t(x,p)$ to post in period t. Optimal vacancies

$$\kappa'(v_t(x,p),x,p) = \beta^f \sum_j \eta_t^j(x,p) W_{t+1}^j(x,p)$$

$$= \beta^f \Pi(\theta_t) \frac{S_t(x,p)}{S_t} \sum_j \frac{S_t^j(x,p)}{S_t(x,p)} W_{t+1}^j(x,p) \tag{D.5}$$

The firm's optimal policy thus depends on the workers' search policies and distribution over labor market states via $S_t(x,p)$, S_t and $S_t^j(x,p)$. It also depends on θ_t which is a function of the other firms' policies and S_t.

Worker Problem   Workers take as given next period's value functions V_{t+1} – and hence the expected surplus of finding a job – as well as the job filling rate $\Lambda(\theta_t)$ and vacancy shares $N_t(x,p)/N_t$ and choose their optimal search effort according to the resulting first order condition.

$$\frac{\mathrm{d}d^j(\ell)}{\mathrm{d}\ell} = \beta \phi_s \Lambda(\theta_t) \left(\sum_{x,p} \frac{N_t(x,p)}{N_t} \max\left\{ V_{e,t+1}^j(x,p), V_{s,t+1}^j(x,p) \right\} - V_{s,t+1}^j(x,p) \right) \tag{D.6}$$

The workers' optimal policies thus depends on the firms' vacancy policies and via $N_t(x,p)$ and N_t. It also depends on θ_t which is a function of the other workers' policies and N_t.

Algorithm   Focus on one skill segment h and let F_t be the distribution of workers across labor market states in period $t = 0, \ldots, T$. The economy is in the initial regime until period $t = -1$. We thus set F_0 equal to the stationary distribution in the initial regime. We assume that the economy has converged to the new regime by period T. All equilibrium objects in period T are thus the equilibrium objects in the stationary equilibrium. The main backward looking object is F_t. Search mass, vacancy mass and tightness can be adjusted instantly and are thus allowed to jump from $t = 0$ to $t = 1$. The distribution F_t only jumps due to non-employability.

   Knowing the initial and terminal stationary equilibrium, we proceed as follows.

1. Guess a sequence $\{F_t^0\}_t$, e.g. a piece-wise linear interpolation between F_T and F_0 taking into account the employability constraint.

2. Set $i = 0$

3. Taking as given the sequence of distributions $\{F_t^i\}_t$ as well as the value functions W_T^j and $V_{s,T}^j$, solve backwards for the equilibrium sequence of policies $\{\ell_t^i, v_t^i\}_t$. Starting with $t = T - 1$, solve for the equilibrium policies in t as follows:

   (a) guess vacancy shares and tightness: $N_t(x, p)$ and θ_t
   (b) solve for optimal search policies $\ell_t^i(j, x, p)$ using equation (D.6)
   (c) update $S_t(x, p)$, S_t, $S_t^j(x, p)$ and θ_t
   (d) solve for optimal vacancy policies $v_t^i(x, p)$ using equation (D.5)
   (e) compute implied vacancy shares and tightness
   (f) if equal to guess, stop, else update guess and go back to (b)
   (g) compute the workers' value: $V_t^j(\sigma) = u^j(\sigma, \ell^i) + \beta \mathbb{E}_{\sigma'|\sigma} \left[V_{t+1}^j(\sigma') | \sigma \right]$
   (h) compute the firm's values: $W_t^j(x, p) = y(x, p) + \beta(1 - \delta_t^j(x, p)) W_{t+1}^j(x, p)$

4. Set $t = t - 1$ and and repeat until $t = 0$

5. Use the transition matrices P_t^i to iterate forward on the distribution starting from F_0 until F_T to get $\{F_t^{i+1}\}_t$

6. Check whether the implied sequence $\{F_t^{i+1}\}_t$ differs from the guess $\{F_t^i\}_t$. Stop if yes. Set $i = i + 1$ and go back to step (3)