Employment and Reallocation Effects of Higher Minimum Wages*

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Abstract

This paper studies the employment and reallocation effects of minimum wages in Germany in a search-and-matching model with worker and firm heterogeneity and multiple employment levels. I find that minimum wages up to 65 to 70 percent of the median wage significantly increase productivity, hours worked and output without reducing employment. In frictional labor markets, however, reallocation takes time whenever the minimum wage cuts deep into the wage distribution. I show that gradually implementing a high minimum wage is necessary to avoid elevated unemployment rates during the transition.

Keywords: Minimum Wage, Reallocation, Employment, Transition Dynamics, Worker and Firm Heterogeneity, Hours Worked, Equilibrium Search-and-Matching Model, Job Search
JEL classification: E24, E25, E64, J20, J31, J38

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1 Introduction

Minimum wages are one of the most popular labor market policies in developed countries to increase low-skill earnings and reduce wage inequality. In light of a growing body of empirical evidence showing that observed minimum wages—ranging between 30 and 60 percent of the pre-reform median full-time wage (Kaitz index[1])—have increased wages and productivity without significantly reducing employment, minimum wages are on the rise (e.g., Cengiz et al., 2019; Dustmann et al., 2022; Dube and Lindner, 2024). Many countries and US states are discussing or have already passed legislation to substantially raise the legal wage floor.[2] However, while our understanding of minimum wage effects has greatly improved, reduced-form analyses of past reforms leave open what to expect from increasing the minimum wage beyond observed levels. At the same time, structural models that can rationalize the empirical evidence and provide guidance for the design of future minimum wage reforms are scarce.[3]

In the context of Germany, this paper studies the short- and long-run employment and reallocation effects of high minimum wages in a rich search-and-matching model that is consistent with the available evidence on observed minimum wage effects (Dustmann et al., 2022). The analysis presents two novel insights. First, high minimum wages of up to 70% of the median can increase output via two-dimensional reallocation towards high-productivity firms and full-time jobs without reducing employment. Second, these long-run effects are the result of a reallocation process that—due to search frictions—takes years to unfold. In particular, the transition to an equilibrium with a high minimum wage can cause significant short-run employment losses if the policy change is implemented abruptly and without advance notice. Transitional unemployment can be avoided, however, if large minimum wage increases are announced in advance and, most importantly, phased in gradually. Hence, in frictional labor markets with residual wage dispersion due to firm heterogeneity, reallocation is a force that mitigates the long-run disemployment effects of high minimum wages and rationalizes why all large minimum wage hikes in the past decades have been implemented over several years.[4]

The analysis is based on a search-and-matching model of the labor market with worker and firm heterogeneity, differences in employment levels (marginal, part-time, full-time), a

[1]The Kaitz index is the ratio of the minimum to the full-time median wage multiplied by 100%. Expressing the minimum wage relative to the median full-time wage facilitates comparisons across time and across countries.

[2]For example, several US states like California, Connecticut, Washington have already increased the minimum to \$15 and other states such as Florida, Illinois and Michigan have started to increase the minimum wage to \$15. In 2016, the UK announced to increase the minimum wage to two thirds of the median wage. The European Union's directive on adequate minimum wages (passed in October 2022), calls to increase minimum wages to 60% of the median wage and many European countries, including Germany, are discussing such minimum wage increases.

[3]Notable exceptions include Engbom and Moser (2022), Berger et al. (2022), Hurst et al. (2022), and Blömer et al. (2024) which I discuss in more detail below.

[4]For example, among the many large state-level minimum wage reforms in the US after the Great Recession, there is only one abrupt minimum wage hike exceeding \$2 (Virginia's first state-level minimum wage of \$9.5 in 2021), and the vast majority (over 97%) of minimum wage changes of at least \$0.25 do not exceed \$1 (Source: Ben Zipperer's minimum wage database). Germany had one major minimum wage increase from €9.8 in 2021 to €10.45 and then €12.0 in the third and fourth quarter of 2022, respectively. The minimum wage increase to a Kaitz index of 67% in the UK was designed to be phased in gradually between 2016 and 2024.

progressive tax-and-transfer system, and endogenous search effort and vacancy posting. In the model, workers receive a fixed piece-rate of match output, making the model's wage equation a structural version of the log-linear AKM wage equation (Abowd et al., 1999), which has been robustly shown to provide an excellent fit to key features of empirical wage dynamics in linked employer-employee data (Card et al., 2013, 2016; Macis and Schivardi, 2016; Bonhomme et al., 2019; Di Addario et al., 2020).[5]

The effect of minimum wages on employment are ambiguous as firms' vacancy posting and workers' job search decisions are affected in opposite directions (Acemoglu, 2001; Flinn, 2006). On the one hand, firms will reduce vacancy creation as the minimum wage cuts into match profits. On the other hand, the minimum wage increases wages, earnings and thus the surplus of finding a job, which leads unemployed workers to exert more search effort. With heterogeneity in firm productivity and employment levels, minimum wages also affect output by changing the composition of jobs, which features inefficiently many low-productivity and low-hours jobs due to search frictions and workers' ability to top-up low-earnings jobs with government transfers. First, raising the minimum wage increases average productivity by pushing low-productivity firms out of the market (Eckstein and Wolpin, 1990; Acemoglu, 2001; Engbom and Moser, 2022). Second, higher minimum wages increase the average employment level, i.e. average hours worked, because workers' incentive to search for jobs with longer hours increases in the hourly wage, which in turn reduces firms' incentives to post vacancies for marginal jobs.[6] This two-dimensional reallocation can lead to significant output gains even if the number of jobs in the economy stays unchanged or decreases slightly.

I estimate the model via the method of simulated moments using German administrative linked employer-employee as well as survey data from the years 2011 to 2014, i.e. the period when the labor market was not yet affected by a federal minimum wage. The model not only matches well the distribution of labor market states and transition probabilities for different demographic groups, but also provides a good approximation to the joint distribution of wages and employment levels and residual wage dispersion. This is important because it determines how many and what kind of jobs are affected by different minimum wage levels, which in turn determines the scope for reallocation effects.

Before analyzing counterfactually high minimum wage levels, I evaluate the introduction of Germany's first federal minimum wage in 2015 through the lens of the estimated model. This large policy shift, which raised the minimum wage from zero to 8.5€ (Kaitz index of 47% in the data and 45% in the model) and affected more than ten percent of jobs, acts as a unique testing ground for the model. I find that the model's short-run predictions of (i) a null-effect on total employment, (ii) a shift from marginal to part-time and full-time jobs, and (iii) an

[5]The piece-rate determines the markdown of wages relative to marginal productivity and is thus the most important parameter for the quantitative employment effects of minimum wages. In the robustness section, I show how the results change for alternative values of the piece-rate.

[6]The minimum wage introduction in Germany in 2015 affected over ten percent of jobs, but only one-third of those were full-time jobs (Dustmann et al., 2022).

increase in average firm productivity are qualitatively and quantitatively consistent with the effects documented by (Dustmann et al., 2022).

The main part of the analysis studies the short- and long-run effects of hypothetical minimum wage reforms that raise the minimum wage beyond observable levels. First, focusing on steady-state comparisons, I find that the total number of jobs does not decrease for minimum wages with Kaitz indices of up to 65–70%, but falls quickly thereafter. At the same time, output grows significantly as the composition of jobs improves with higher minimum wages. At a Kaitz index of 65%, just before employment starts to fall, average firm productivity, total hours worked and total output are 4.6%, 5.0%, and 4.2% above their respectice baseline levels. Due to these improvements in job composition, output continues to increase up to a Kaitz index of 75% even though the number of jobs declines.

Second, I show that the favorable long-run effects of high minimum wages are the result of a potentially painful transition process—depending on the design of the reform. To illustrate the role of search frictions, I first study the case where the minimum wage is abruptly implemented starting from the baseline equilibrium without a minimum wage. For sufficiently high minimum wages, i.e., with Kaitz indices above 50%, such a large and sudden increase in the minimum wage leads to a discrete drop in employment as firms lay off workers whose jobs have become unprofitable. In the presence of search frictions, it takes years until employment recovers. Besides the initial spike in unemployment, slow convergence is driven by the fact that a significant share of laid-off workers slide into long-term unemployment where search efficiency and hence job finding rates are lower. For example, when switching from a minimum wage of zero to a minimum wage with a Kaitz index of 65%— which will not decrease the steady state employment rate—employment drops by 4.2% in the year of the reform and is still about 1.5% and 0.9% below baseline in years two and five after the minimum wage hike.

Note that the disconnect between short- and long-run employment effects is mainly driven by the share of jobs in the initial equilibrium that will become unprofitable and will thus be destroyed as the minimum wage is introduced. For low minimum wages with Kaitz indices below 50%, the share of unprofitable jobs is close to zero even though a substantial share of jobs pays less than the minimum wage. This non-linearity implies that (i) initial job destruction and transiitonal unemployment are quantitatively insignificant for low and moderate minimum wages, and (ii) large minimum wage hikes starting from low and moderate minimum wages still lead to substantial transitional employment losses.

I then analyze how the design of minimum wage reforms affects the dynamics of employment effects. I find that transitional employment losses can be avoided when minimum wage hikes are gradually phased in over several years and the full sequence of reforms is announced in advance. As firms anticipate future minimum wage increases, they immediately adjust their vacancy posting. Hence, the process of reallocation toward more productive firms starts even before the minimum wage reaches its target level. This substantially reduces the share of unprofitable jobs at every step of the reform. For example, gradually raising the minimum wage to a Kaitz index of 65% over a period of five years reduces the maximum employment loss along the transition by 87.0% (from 4.2% to 0.5%).[7]

Besides the design of minimum wage reforms, the speed of convergence is also affected by the degree of labor market turnover in the baseline equilibrium. In contrast to Anglosaxon and Nordic countries, but in line with other Western European countries, the German labor market is characterized by relatively low labor market turnover (Hobijn and Şahin, 2009). This naturally leads to slower convergence. While convergence is substantially faster in a re-parameterized version of the model where baseline job finding rates are doubled, large and abrupt minimum wage hikes still elevated unemployment in the first five years after the reform. The reason is that initial job destruction is not affected by the degree of labor market turnover. Hence, gradual phase-ins are still necessary to avoid transitional unemployment even in a high-turnover labor market.

Finally, I also analyze the welfare effects of higher minimum wages. I find that welfare effects vary significantly across different demographic groups of workers. While average welfare increses by up to 5.9% in terms of consumption equivalents, single women with kids experience welfare losses despite higher wages and earnings. The main reason is that the decline in vacancies for marginal jobs and the resulting increase in hours worked can partially offset the utility gains from higher wages and consumption. As single women with kids have a stronger disutility for full-time jobs, they are disproportionately affected by the reduction in low-hours vacancies. Heterogeneity in employment levels is thus not only relevant for output effects but also add nuance to our understanding of welfare effects of minimum wages.

Related Literature. This paper makes several contributes to the literature on minimum wages. Relative to the surge in reduced-form analyses of minimum wage reforms (Dube, 2019; Dube and Lindner, 2024), structural analyses of minimum wage effects are scarce.

Several recent papers study minimum wage effects in neoclassical frameworks without frictional unemployment. Ahlfeldt et al. (2021) and Bamford (2021) study the employment and welfare effects of minimum wages using a spatial equilibrium model with monopsonistic labor markets estimated using German data. Berger et al. (2021) study the (long-run) welfare and efficiency effects of minimum wages in the US using a model with oligopsonistic labor markets and two-sided heterogeneity. While they find positive welfare gains, they find only small efficiency gains of higher minimum wages.[8]

[7]The gradual reform first increases the minimum wage from zero to a Kaitz index of 45% and then gradually increases it to 65% over the next five years. When starting out in an equilibrium with a Kaitz index 45% instead of zero, the maximum employment loss along the transition is only 0.2%.

[8]The muted output effect in Berger et al. (2021) results from reallocation *down* the productivity ladder as low productivity firms, whose wages increase because of the minimum wage and which some workers favor due to idiosyncratic preferences, now attract more workers. This mechanism is also (partly present) in my model with on-the-job search as workers stay longer with low-productivity firms because wage differentials are compressed. However, this mechanism is dominated by reduced vacancy posting of low-productivity firms. A key difference is that, in the random search setup in this paper, workers cannot target their labor supply to specific firms for which they have non-pecuniary preferences.

This paper is more closely related to the strand of the literature that studies minimum wage effects in the context of search-and-matching models. Compared to early theoretical and structural research that uses rather stylized models, I adopt a more quantitative approach that leverages administrative matched employer-employee data (e.g. Burdett and Mortensen, 1998; van den Berg and Ridder, 1998; Bontemps et al., 1999; Eckstein and Wolpin, 1990; Acemoglu, 2001; Flinn, 2006).

Two more recent studies also analyze minimum wage effects using quantitative search-and-matching models with two-sided heterogeneity. Engbom and Moser (2022) estimate a wage-posting model in order to quantify the contribution of an observed increase in the minimum wage to the decline of wage inequality in Brazil. While they focus on the distributional effects of the minimum wage, their paper also highlights the role of reallocation towards high-productivity firms. For the particular Kaitz index of 55% studied in their paper, my model yields very similar effects on average productivity (3%), but a larger increase in output (2.3% vs. 1%) as they estimate a stronger decline in employment (no change vs. 0.7%) and do not consider changes in average hours worked. For Germany, Blömer et al. (2024) estimate the wage posting model by Bontemps et al. (1999) to analyze the effects of minimum wages on full-time employment. They find small employment effects for minimum wages below 12€, but extremely large disemployment effects in East Germany for minimum wages above 5€.[9] However, their analysis is restricted to full-time workers, which in Germany accounted for only a third of all jobs affected by the minimum wage introduction in 2015. In contrast to Engbom and Moser (2022) and this paper, Blömer et al. (2024) do not investigate productivity and output effects.

The first key contribution of this paper is that I study employment effects not only at the extensive but also at the intensive margin, taking account of the fact that a large share of minimum wage jobs in Germany are marginal or part-time jobs. The analysis thus allows for output effects through two-dimensional reallocation, i.e., toward high-productivity firms and away from low-hours jobs, as documented by Dustmann et al. (2022). Both channels are found to be quantitatively important for output effects. The welfare analysis shows that, in the presence of intensive employment effects, the positive welfare effects related to higher wages and consumption can be attenuated from higher disutility of working longer hours.

My model also differs from these papers in that it includes endogenous search effort, a progressive tax-and-transfer system and a different wage setting protocol. Endogenizing search effort allows workers to react to minimum-wage induced changes in the surplus of working or in the surplus of changing employers or work longer hours. A more realistic tax-and-transfer system affects the scope for reallocation as skill-independent subsistence benefits subsidize low-earnings jobs leading to disproportionately many low-hours and low-productivity jobs in the lower skill segments. The assumption that workers earn a fixed piece-rate of match output has the desirable implication that the model's wage equation is a structural representation of the empirically

[9] The large heterogeneity in employment effects across regions is at odds with the empirical evidence (Dustmann et al., 2022).

highly successful AKM wage equation (Abowd et al., 1999; Card et al., 2013), allowing for a tight link between model and data during estimation.

The second key contribution is that I investigate the transition dynamics of employment and reallocation effects of different minimum wage reforms. While the notion of reallocation effects is not new, this paper is the first to study how, in frictional labor markets with heterogeneous firms, reallocation effects shape the transition path of employment effects and clarifies that the details of the implementation, i.e., the size of the minimum wage hike relative to the initial minimum wage level and the phase-in period, crucially shape the dynamics of employment effects. The model's predictions are consistent with the (scarce) empirical evidence on dynamic employment effects of minimum wages. In the US, Cengiz et al. (2019) do not find significant differences between short- and long-run effects of minimum wage reforms which, on average, increased the minimum wage by only 10% and have terminal Kaitz indices below 60%. Clemens and Strain (2021) study the employment effects of gradual, state-level minimum wage increases between 2014 and 2019 and find that (i) the disemployment effects of these combined reforms (relative to the last period before the reform) increase in the first part and then level off at the end of the implementation window; and (ii) large reforms have disproportionate disemployment effects. My model rationalizes both findings.[10] For the German minimum wage introduction, Dustmann et al. (2022) show that reallocation toward more productive firms takes time as workers are substantially more likely to work at a high-productivity firm two years after the reform than immediately after the reform. Again, the corresponding reallocation effects in the model align both qualitatively and quantitatively with these empirical findings.

The only other paper that studies both short- and long-run effects of minimum wages in a structural model is the paper by Hurst et al. (2022) who study the effects of the $15 minimum wage proposal in the US using a directed search model where homogeneous firms operate a putty-clay technology and hire low- and high-skill workers. Interestingly, they reach the opposite conclusion that minimum wages have large disemployment effects in the long run and moderate disemployment effects in the short-run as firms slowly substitute high-skill labor and capital for low-skill labor every time they are allowed to adjust their capital stock. I view these papers as highly complementary: while their analysis focuses on sluggish adjustment of inputs within homogeneous firms, this paper studies reallocation across firms with heterogeneous productivity.

This paper is also related to the vast empirical literature evaluating past reforms (e.g. Cengiz et al., 2019; Dustmann et al., 2022).[11] The analysis rationalizes the "elusive employment effect" (Manning, 2021), i.e. the finding that past minimum wages (up to a Kaitz index of 60%) have not had a significant impact on total employment (positive or negative). In particular, the model generates a very small effect on total employment for all previously observed minimum wage levels as the net effect of workers' search and firms' vacancy responses.[12] In addition, this

[10] Unfortunately, there is no empirical analysis of the dynamic effects of large and abrupt minimum wage increases.

[11] Empirical studies of the German minimum wage introduction in 2015 include Garloff (2016); vom Berge et al. (2016); Bossler and Gerner (2016); Caliendo et al. (2017); Ahlfeldt et al. (2018); Dustmann et al. (2022).

[12] See Cengiz et al. (2019) for evidence on this null-effect on total employment independent of the minimum wage level (below a Kaitz index of 60%). See Dube (2019) for a review of the empirical minimum wage literature.

paper quantitatively rationalizes the reallocation patterns away from marginal jobs and towards more productive firms observed following the German minimum wage introduction in 2015 (e.g. Garloff, 2016; Dustmann et al., 2022).

Outline. The remainder of the paper is structured as follows. Section 2 presents the equilibrium search-matching model. Section 3 describes the estimation procedure and evaluates how the model fits the pre-reform data. Section 4 analyzes the introduction of the German minimum wage and compares the model's predictions to the findings of the empirical literature. Section 5 analyzes the short- and long-run effects of counterfactually high minimum wages. Finally, section 6 concludes and discusses areas for future research.

2 Model

I study an economy where a unit mass of workers meet a mass m_f of firms in a labor market with search frictions. Time is discrete and both workers and firms are infinitely-lived. Workers differ by human capital and demographics, and firms differ by productivity and offer jobs with different hours requirements. Both worker and firm heterogeneity is exogenous and time-invariant.

2.1 Workers

Workers differ by gender and family status. In particular, I distinguish between the following five demographic groups indexed by j: married men, single men, single women with and without kids, and married women. Let P_j denote the population share of group j. These demographic groups of workers face different tax-and transfer schedules and differ in terms of their preferences over employment levels.[13] Heterogeneity in the disutility of hours worked is an important determinant of observed differences across demographic groups and ensures that a aggregate differences in working hours are not purely frictional.

Workers further differ by their time-invariant human capital (skill) h. The gender-specific distribution function of human capital is $\Phi^{g(j)}$ where g is the gender of group j. I assume that the labor market is segmented with respect to workers' skill levels such that there is a continuum of independent labor markets – one for each level of h (van den Berg and Ridder, 1998; Engbom and Moser, 2018). In the remainder of this section, I will often suppress the dependence on j and h for the sake of brevity.

A worker can be employed, $s = e$, short-term unemployed, $s = su$ or long-term unemployed, $s = lu$. There are three employment levels (hours worked), x, which I label full-time, $x = ft$, part-time, $x = pt$ and marginal employment, $x = mj$.[14] The parameters e_x denote hours worked in full-time, part-time and marginal jobs respectively. In addition, jobs differ with respect to

[13] As men with and without children are similar with respect to all targeted moments, I only distinguish between single and married men. The same holds for married women. Appendix Table A.1 shows the population shares of each demographic type.

[14] Marginal employment is referred to as "mini-jobs" in Germany. The monthly income of a mini-job is 450€ or less and not subject to personal income taxation.

the employer's productivity p which will be described below. While short-term unemployed workers receive unemployment insurance proportional to their previous earnings, all long-term unemployed workers receive the same unemployment benefits, i.e., a subsistence minimum. In sum, for each skill level h, there is a continuum of idiosyncratic states for employed and short-term unemployed workers and a single state for long-term unemployment. Formally, the space of endogenous labor market states of a worker is

$$\mathcal{S} = \Big\{ \big\{ (s, x, p) \mid s \in \{e, su\}, x \in \{ft, pt, mj\}, p \geq 1 \big\}, lu \Big\}$$

In the following, I denote by σ a point in the state space and by F the distribution of these endogenous labor market states (given j and h).

Workers exert costly search effort ℓ to find (better) jobs. A worker in employment state σ meets a vacancy with probability $\lambda_\sigma(\ell) = \phi_\sigma \ell \Lambda(\theta)$, where labor market tightness θ is taken as given and ϕ_σ captures search efficiency. I assume that search efficiency differs by employment level, e_x, and between short- and long-term unemployed ($\phi_{su}, \phi_{lu}, \phi_{e_x}$) in order to match observed job-to-job transition rates and job finding rates out of short- and long-term unemployment. The difference in search efficiency between short- and long-term unemployed workers will become important when studying transition dynamics of minimum wage reforms. Importantly, not every meeting results in a match since search cannot be directed toward certain employment levels or high-productivity firms, and workers may decline lower-valued offers.

As there is no savings device, consumption $c^j(\sigma)$ of a worker in labor market state σ equals her net income, which is the sum of net earnings, $T^j(y(\sigma))$, and transfer income $B^j(\sigma)$. Married workers additionally receive non-labor income, y^j_{free}, which captures the effect of a partner's income on the consumption of married workers. Depending on a workers employment status, government transfers consist of unemployment insurance for short-term unemployed workers, skill-independent subsistence benefits for long-term unemployed workers, and top-up transfers for employed workers with sufficiently low earnings.

Apart from consumption, workers' utility is also a function of search effort ℓ and the job type x:

$$u^j(\ell, \sigma, h) = \tilde{u}(c^j(\sigma, h)) - d(\ell) + \nu^j\big(x(\sigma, h)\big) \tag{1}$$

Here, $\tilde{u}(c)$ is a concave flow utility function of consumption, $d(\ell)$ is the convex cost of search effort and $\nu^j(x(\sigma, h))$ captures the (dis-)utility of different employment levels relative to nonemployment. The latter may depend on demographics in order to capture that, for example, single women with kids may have a stronger dislike for full-time work compared to men or single women without children.[15]

[15] I emphasize that these "preference" parameters not only capture the tastes for leisure, but also exogenous constraints such as childcare obligations. As I do not explicitly model policies affecting child care constraints, using such a proxy is justified even though the parameter is not policy-invariant outside the model.

2.2 Firms

Firms are risk-neutral and have the same discount factor as workers, $\beta_f = \beta$. Firms are owned by pure capitalists with linear utility who are outside the model. Hence, workers do not receive a share of firms' profits. Firms are heterogeneous in terms of their time-invariant productivity $p \sim \Gamma$ and can employ workers of all skill levels h and at all employment levels x. Firms operate a linear production technology such that total output of a firm with productivity p is the sum of all its match outputs

$$\sum_x \int_{\underline{h}}^{\bar{h}} f(h, x, p) L(h, x, p) \mathrm{d}h \tag{2}$$

where $f(h, x, p)$ is the flow output produced by workers in skill segment h working a type-x job at a firm with productivity p, and $L(h, x, p)$ is the mass of such jobs in the respective firm. This assumption—which is rather standard in the search-and-matching literature—implies that there are no complementarities between low- and high-skill workers.[16]

Each period and for all job types x, firms post vacancies to attract workers. Vacancies may result in an employment relationship starting in the subsequent period. In doing so, firms incur convex costs $\kappa(x, v)$ for each job type x. This ensures that all job types will be offered in equilibrium. Unfilled vacancies are not carried over to the next period but have to be re-posted. Combined with linear production, this implies that hiring a worker does not affect future recruitment and that firms will not reject workers of a particular demographic type even if different workers are more or less likely to switch employers than others. While production is linear, the convexity in the vacancy posting cost function puts a bound on the number of vacancies that firms are willing to post and thus on its number of employees.[17]

2.3 Labor Market

The total mass of search, S, and the total mass of vacancies, N, determine the total number of matches through the matching function $M = N^{\xi} S^{1-\xi}$, where ξ is the elasticity of matches with respect to the mass of posted vacancies.[18] Labor market tightness is defined as $\theta = N/S$. Hence, the aggregate contact rates for a unit of search and a vacancy are $\Lambda(\theta) = \theta^{\xi}$ and $\Pi(\theta) = \theta^{\xi-1}$, respectively. In addition, let $\Psi(p, x)$ denote the joint distribution of firm productivities and job types among all vacancies in a given skill segment.

Employment relationships are terminated for three mutually exclusive reasons. First, workers may voluntarily change firms and/or employment levels as a result of on-the-job search. In equilibrium, firms with low productivity will be more likely to experience this event. Second,

[16] Among others, this assumption is also made in Bagger et al. (2014), Bagger and Lentz (2018), or Engbom and Moser (2022).

[17] The advantage of this setup, i.e., linear production and convex vacancy posting costs, is that the firm's current number of employees and their distribution across heterogeneous worker types is not a state variable, which makes the model computationally tractable.

[18] The formulas for S and N are given in Appendix B.

workers may be hit by a so-called "Godfather shock", which forces them to switch to a different job that is randomly drawn from the distribution of vacancies (Jolivet et al., 2006).[19] This is a standard feature to account for the substantial share of job-to-job transitions that are accompanied by a wage cut and cannot be explained by on-the-job search. Without Godfather shocks, workers in the model would never move down the job ladder, thereby distort the distribution of workers across heterogeneous firms—a key statistic for the study of reallocation effects. The Godfather shock arrives with exogenous probability $\pi_{e|e_x}$ and captures involuntary and unintended job-to-job transitions unrelated to workers' search effort. Third, matches can be destroyed such that the worker transitions into short-term unemployment. This may happen either exogenously with probability $\pi_{su|e_x}$, or endogenously whenever a minimum wage hike makes the match unprofitable, which forces the firm to lay off the worker.

2.4 Wage Setting

When a worker is employed in a type-x job at a firm with productivity p, the match output is $f(h, x, p) = e_x hp$. I assume that workers receive a constant and exogenous share $r \in (0, 1)$ of match output such that gross earnings equal $y(h, x, p) = \max\{rf(h, x, p), \bar{w}e_x\}$, where $\bar{w}$ is the minimum hourly wage. Note that the piece-rate r pins down the aggregate labor share, but not the aggregate profit share in the economy. This is because firms have to use a significant share of their revenue net of wage payments in order to cover the costs of vacancy creation. More importantly, the piece-rate is equal to the inverse of the wage markdown relative to workers' marginal product, $1 - r$, and thus captures the degree of labor market power in a transparent way.

The assumption of a fixed piece-rate implies that wages do not depend on workers' and firms' outside options through bargaining or reservation wages. Instead, in the absence of a minimum wage, log wages are linear in log worker ability and log firm productivity. The model's wage equation is thus a structural counterpart to the simple yet remarkably successful two-way fixed effect AKM model (Abowd et al., 1999), which has become the workhorse model in labor economics to study the structure and dynamics of wages.

Several key implications of the AKM model have been shown to hold in various linked employer-employee datasets. First, firms pay the same proportional wage premium to newly hired workers irrespective of their wages at previous employers (Card et al., 2013, 2016; Macis and Schivardi, 2016; Di Addario et al., 2020). Hence, the data suggest that a major component of a worker's outside option—the wage premium or productivity of their current employer—has almost no impact on their wage after a job-to-job transition to another firm. Similarly, the model's implication that wages do not depend strongly on workers' outside options is further supported

[19] Unintended job-to-job transitions may be the result of firms' outplacement programs, workers' search effort after an advance-notice layoff or family-related events that force workers to move and look for a new job immediately.

by the recent finding of Jäger et al. (2020) who show that—even for previously unemployed workers—wages are unaffected by UI-induced shifts in the value of non-employment.[20]

Second, firms offer the same proportional wage premium to their all of their workers independent of workers' ability. The fact that low-ability workers do not earn lower wage premia at high-prductivity firms is consistent with the additive structure of the AKM wage equation (Card et al., 2013, 2016; Macis and Schivardi, 2016; Bonhomme et al., 2019; Di Addario et al., 2020).

Third, the available evidence on spillover effects of minimum wages suggests that wage bargaining has not been an important mechanism behind observed wage effects. While (Cengiz et al., 2019) find modest wage spillovers in the US, they are confined to incumbent workers. The fact that new hires do not experience wage spillovers suggests that the value of outside options or reservation wages of nonemployed workers due to an increase in the minimum wage do not play a significant role in wage setting.[21]

In sum, the assumptions of a fixed piece-rate and log-linear production function and the implied log-linear AKM-style wage equation not only allow for a clean mapping between model and empirical moments when estimating the model using linked employer-employee data, but also—in contrast to bargaining models—squares well with recent empirical evidence on wage dynamics and minimum wage effects.

2.5 Worker Problem

Each period, workers choose search effort ℓ and reject or accept job offers in order to maximize discounted lifetime utility while taking labor market tightness and the distribution of vacancies as given.

The value of long-term unemployment for a type-(j,h) worker solves the following Bellman equation:

$$V_{lu} = \max_{\ell} \Big\{ u\big(\ell, lu\big) + \beta\lambda_{lu}(\ell)\mathbb{E}_{(x,p)}\Big[\max\big\{V_e(x,p), V_{lu}\big\}\Big] + \beta\big(1-\lambda_{lu}(\ell)\big)V_{lu}\Big\} \tag{3}$$

Search effort ℓ is associated with lower flow utility but a higher probability of meeting a firm. Upon meeting a firm offering a (x,p) job, the worker accepts the job if and only if the value of the employment relationship, $V_e(x,p)$, exceeds the value of remaining long-term unemployed. The

[20]The insensitivity of wages to workers' outside options cannot be rationalized by bargaining models unless workers have unrealistically high bargaining power. Wage-posting models have similar difficulties in explaining the insensitivity of wages to the value of non-employment if reservation wages vary significantly with unemployment benefits. However, it must be noted that direct evidence on the role of outside options on wage setting is scarce. In fact, the study by Jäger et al. (2020) is the only direct evidence of whether UI benefits affect wage setting. Indirect evidence on the matter is mixed. A couple of papers find evidence consistent with wage effects of outside job offers (Staiger et al., 2010; Beaudry et al., 2012). In contrast, the finding of Chodorow-Reich et al. (2019) that the substantial increase in UI benefits during the Great Recession did not cause significant reductions in vacancy creation due to higher wage demands is inconsistent with standard bargaining models if the value of non-employment is largely driven by unemployment benefits.

[21]Cengiz et al. (2019) suggest that spillovers among incumbents are best explained by relative pay concerns within firms.

expectation is taken with respect to the distribution of vacancies in the worker's skill segment. With probability $1 - \lambda_{lu}(\ell)$, the worker does not meet a firm and remains long-term unemployed.

The value of short-term unemployment when the previous job was of type x at a type-p firm is given by:

$$
\begin{aligned}
V_{su}(x,p) = \max_{\ell} \Big\{ & u\big(\ell,(su,x,p)\big) + \beta \pi_{lu|su} V_{lu} \\
& + \beta \lambda_{su}(\ell) \mathbb{E}_{(x',p')} \Big[\max \big\{ V_e(x',p'), V_{su}(x,p) \big\} \Big] \\
& + \beta \big(1 - \pi_{lu|su} - \lambda_{su}(\ell)\big) V_{su}(x,p) \Big\}
\end{aligned} \tag{4}
$$

The only difference to long-term unemployment is that the worker transitions from short- to long-term unemployment with exogenous probability $\pi_{lu|su}$ and receives unemployment benefits that depend on the worker's previous earnings.

The value of a worker employed at a type-p firm on a type-x job is

$$
\begin{aligned}
V_e(x,p) = \max_{\ell} \Big\{ & u\big(\ell,(e,x,p)\big) + \beta \pi_{su|e_x} V_{su}(x,p) \\
& + \beta \lambda_{e_x}(\ell) \mathbb{E}_{(x',p')} \Big[\max \big\{ V_e(x',p'), V_e(x,p) \big\} \Big] \\
& + \beta \pi_{e|e_x} \mathbb{E}_{(x',p')} \Big[V_e(x',p') \Big] \\
& + \beta \big(1 - \pi_{su|e_x} - \lambda_{e_x}(\ell) - \pi_{e|e_x}\big) V_e(x,p) \Big\}
\end{aligned} \tag{5}
$$

Employed workers become short-term unemployed with probability $\pi_{su|e_x}$, receive a job offer that they can decline through on-the-job search with probability $\lambda_{e_x}(\ell)$ and are involuntarily reallocated to a different job with probability $\pi_{e|e_x}$.

All workers may have an incentive to search for a (better) job. Given labor market tightness and the distribution of vacancies, the first order condition determining optimal search effort is given by

$$
\frac{\mathrm{d}d(\ell)}{\mathrm{d}\ell} = \beta \frac{\partial \lambda_\sigma(\ell|\theta)}{\partial \ell} \Bigg(\underbrace{\mathbb{E}_{(x,p)} \Big[\max \big\{ V_e(x,p), V(\sigma) \big\} \Big] - V(\sigma)}_{\text{expected surplus of meeting a firm}} \Bigg) \tag{6}
$$

2.6 Firm Problem

Firms maximize expected discounted profits taking as given labor market tightness, the distribution of vacancies and the distribution of workers' search effort. As total production is additive in h and x, each firm faces a sequence of independent optimization problems—one for each (h,x)-segment.

The value $W^j(x,p)$ of a type-x employment relationship with a worker of type j for a firm with productivity p is given by

$$W^j(x,p) = \frac{(1-\bar{r})f(x,p)}{1-\beta^f\big(1-\delta^j(x,p)\big)} \tag{7}$$

where $\delta^j(x,p)$ is the probability that the employment relationship ends, either due to exogenous job destruction, a Godfather shock or on-the-job search, and $(1-\bar{r})f(x,p)$ is revenue net of wage payments with $\bar{r} > r$ if the minimum wage is binding. When posting a vacancy, the firm has to take the expectation over worker types as they differ in their on-the-job search effort, which affects the separation probability and expected value of a match.

Optimal vacancy posting requires firms to post vacancies until the marginal cost of posting another vacancy is equal to the discounted expected value of an employment relationship weighted by the probability of filling the vacancy. Hence, the firm's optimality condition is given by:

$$\kappa'(v,x) = \beta^f \eta(x,p)\mathbb{E}\big[W^j(x,p)\big] \tag{8}$$

where $\mathbb{E}[W^j(x,p)]$ is the expexted value of an employment relationship, and $\eta(x,p)$ is the probability of filling a vacancy.[22]

2.7 Equilibrium

In each skill segment, a stationary equilibrium consists of value functions, V^j_{lu}, $V^j_{su}(x,p)$, $V^j_e(x,p)$, search effort policy functions, $\ell^j(\sigma)$, vacancy posting policy functions, $v(x,p)$, labor market tightness, θ, a distribution of vacancies, $\Psi(x,p)$, and a distribution of workers across states, $F^j(\sigma)$, that satisfy the following conditions. First, given labor market tightness and the distribution of vacancies, the value and search effort policy functions solve the workers' problem (equations 3, 4, 5, and 6). Second, given labor market tightness, workers' search policies and value functions, and the distribution of workers across states, firms' vacancy posting policy functions solve the firms' optimality conditions (equation 8). Third, the distribution of workers across states is stationary. That is, given the economy starts at this distribution and given the policy functions and labor market tightness, the distribution of workers across states will not change.

2.8 Minimum Wage Effects

As wages are marked down relative to productivity, minimum wages do not necessarily reduce employment. Instead, their impact on employment is ex-ante ambiguous and depends on the relative importance of several forces that are present in frictional labor markets.

Without heterogeneity across workers, firms and job types and in the absence of on-the-job-search, there are two broad channels. On the one hand, minimum wages affect labor demand, i.e., firms' vacancy posting decisions. Higher minimum wages reduce flow profits and thus the value of

[22]The probability of filling a vacancy equals the aggregate contact rate times the share of searching workers willing to accept the offer. The formulas for $\delta^j(x,p)$, $\eta(x,p)$ and $\mathbb{E}[W(x,p)]$ are given in Appendix B.

creating a vacancy. This reduces the number of vacancies and thus employment. However, when many firms reduce vacancy posting congestion for firms declines, which increases the probability of filling a vacancy and mitigates the negative effects on vacancy posting. On the other hand, minimum wages affect labor supply, i.e., workers' search effort and job offer acceptance decisions. Higher minimum wages increase the surplus of employment which increases search effort, job finding rates and employment. However, increased search effort also increases congestion which lowers job finding rates and employment. Obviously, both channels are intertwined and the net effect on the number of jobs is ambiguous.

Adding heterogeneity also changes the composition of jobs. With firm heterogeneity, a given minimum wages hits flow profits of low-productivity firms harder than those of high-productivity firms, which leads to asymmetric vacancy responses. It may even be the case that the expected value of posting a vacancy increases for high-productivity firms, for example because the reduction in vacancy posting by low-productivity firms reduces congestion while flow profits of high-productivity firms are hardly affected by the minimum wage. The asymmetric vacancy response increases the share of high-productivity vacancies which may affect workers' expected surplus of employment and hence search effort. With on-the-job search, the minimum wage reduces differences in match durations across firms as wage differences between low- and high-productivity firms decrease. This attenuates the decline in vacancy posting for low-productivity jobs as they are able to retain workers for longer.

With multiple employment levels, the minimum wage increases the surplus of full-time jobs relative to part-time (and marginal) jobs because the earnings difference increases whereas the disutility of hours worked remains unchanged. This reduces match durations for part-time jobs and shifts the distribution of vacancies towards full-time jobs. With heterogeneity in workers' disutility of hours worked, the change in the composition of vacancies will lead to asymmetric search effort responses and thus affect the composition of search in a way that workers with a lower disutility of hours worked contribute more to the aggregate search mass. This amplifies the shift towards full-time vacancies as match durations of part-time jobs decline. In addition, workers with a higher disutility of hours worked will remain unemployed for longer.

3 Estimation

In this section, I first describe the pre-set parameters and parameterize workers' flow utility and skill distributions, firms' productivity distribution and vacancy posting cost function and the tax schedule (section 3.1). Second, I discuss which moments I target in the method of simulated moments in order to identify the jointly estimated parameters (section 3.2). Third, I evaluate how well the model fits important moments in the data and show which parameters drive these moments in the model (section 3.3).

3.1 Parameterization and Pre-Set Parameters

One model period corresponds to one quarter. I set the quarterly discount factor of both workers and firms to $\beta = 0.985$ and choose the initial minimum wage in Germany of 8.5€ as the numéraire (in 2015 Euros).

Employment Levels. The employment level for full-time employment, e_{ft} is normalized to one and e_{pt} and e_{mj} are set to match the ratio of average weekly hours of part-time and marginal workers relative to full-time employed workers reported by Dustmann et al. (2022) who have access to hours worked in the German social security data. This yields $e_{pt} = 0.615$ and $e_{mj} = 0.223$.

Taxes & Transfers. The German transfer system distinguishes between short- and long-term unemployment. During the first year of unemployment, workers are paid a fixed fraction b of their previous earnings, but not less than the subsistence minimum B_{min}. Long-term unemployed workers receive the subsistence minimum B_{min} independent of their previous earnings. Employed workers are eligible for transfers to top up their earnings if their net earnings are sufficiently low. In particular, if earnings are below B_{min}/τ_{top}, they receive transfers B_{min} and are allowed to keep $1 - \tau_{top}$ of their net earnings. Finally, married workers receive non-labor income y^j_{free} which is always deducted from B_{min}.[23] Hence, subsistence benefits for type-j workers may not exceed $B^j_{min} = \max\{B_{min} - y^j_{free}, 0\}$. In sum, a type-$j$ worker with skill h faces the following consumption schedule

$$c^j(h,\sigma) = \begin{cases} y^j(h,x,p) + \max\left\{B^j_{min} - \tau_{top} y^j(h,x,p), 0\right\} + y^j_{free} & \text{if } s = e \\ by^j(h,x,p) + \max\{B^j_{min} - by^j(h,x,p), 0\} + y^j_{free} & \text{if } s = su \\ B^j_{min} + y^j_{free} & \text{if } s = lu \end{cases} \tag{9}$$

where $\sigma \in \mathcal{S}$ denotes one state in the worker's state space.

In accordance with the tax-and-transfer system in Germany, I set the replacement rate during short-term unemployment b to 0.6, the monthly subsistence transfers B_{min} to 800€ (55% of of full-time monthly earnings at the minimum wage of 8.5€), and the top-up parameter τ_{top} to 0.8. Using SOEP data that allow me to link spouses, I calculate average net earnings of the spouses of the married men and women in my sample. I then assign half of that amount to the spouse as non-labor income y^j_{free}.[24] Regarding labor income taxes, I assume that workers pay a constant marginal tax rate τ^j on earnings above an exemption level D^j that varies across demographic groups. The parameters of the tax schedule are estimated on SOEP data for gross

[23] The type-specific and exogenous non-labor income y^j_{free} represents a share of the partner's income for married workers. Singles do not receive such non-labor income.

[24] On average, married women have roughly 894€ and married men 409€ in non-labor income from their spouses' net earnings. With $B_{min} = 800$, this implies that married women are not eligible for subsistence benefits and married men receive at most half of total subsistence benefits. Singles are assumed to have no non-labor income and are hence eligible for the full amount of subsistence benefits.

and net earnings for the years 2013 and 2014, separately for different the socioeconomic worker types. Appendix Figure A.1 shows that the estimated average tax function provides a good fit to the binned data.

Productivity Distributions I assume that firm productivity $p \geq 1$ is drawn from a Log-Gamma distribution with shape and scale parameters α_p and θ_p. Human capital is drawn from a gender-specific left-truncated Log-Normal distribution defined by μ_h^g and σ_h^g where g denotes the gender of type-j workers. The truncation bound h_{min} is chosen such that the lowest possible wage—resulting from a match between the least productive firm ($p_{min} = 1$) and lowest skilled worker—generates a wage of 4€ i.e. $rh_{min}p_{min} = 4$.[25]

Preferences Workers' utility depends on consumption, job search and the employment level in the following way:

$$u^j(h, \ell, \sigma) = \frac{c^j(h, \sigma)^{1-\gamma_c}}{1-\gamma_c} - \ell^\zeta + h^\epsilon \sum_x \gamma_x^j \mathbb{1}\{x(\sigma) = x\} \tag{10}$$

where the CRRA parameter $\gamma_c \geq 0$ captures the degree of concavity for consumption utility, $\zeta > 1$ the degree of convexity of search disutility, the preference shifters $\gamma_x^j \in \mathbb{R}$ determine the (dis-)like for the different employment levels (relative to nonemployment) for type-j workers, and $\epsilon \in \mathbb{R}$ determines how the preference shifters scale with human capital h.

Vacancy Posting I further assume that the cost of posting v vacancies for type-x jobs in skill segment h is given by

$$\kappa(v, h, x) = e_x \kappa_1 v^{\kappa_2} f_h(h)^{1-\kappa_2} \tag{11}$$

where $\kappa_1 > 0$ and $\kappa_2 > 1$ determine the slope and convexity of the cost function, f_h is the density of workers' human capital and e_x is the employment level.[26]

Matching Function The vacancy-elasticity of the matching function, ξ, is set to 0.3 following the literature review by Petrongolo and Pissarides (2001).[27] Hence, a reduction in the number of vacancies by 10% reduces the number of matches by 3%. In the robustness analysis, I show that the main insights on employment and reallocation effects of counterfactually high minimum wages are robust to assuming larger values of the vacancy-elasticity of matches. However, increasing ξ to 0.5 implies counterfactually large disemployment effects for the German minimum wage

[25] Data from the SOEP as well as the German Survey of Earnings Structure show that there are virtually no jobs with an hourly wage below 4€ (Minimum Wage Commission, 2018).

[26] I scale the cost of posting vacancies by the density of human capital due to the assumption of segmented labor markets.

[27] Engbom and Moser (2022) use $\xi = 0.5$, but note that this is too high relative to the empirical evidence such that their disemployment effects represent lower bounds.

introduction in 2015 as well as for moderate minimum wage levels that have been implemented and analyzed in other countries (Dube, 2019).

Wage Setting I set the piece rate of output workers receive to $r = 0.63$ in order to match the aggregate labor share in Germany between 2011 and 2014. Recall that this does not imply a pure profit share of 37% as firms have to use some of their revenue in order to cover vacancy posting costs. In the estimated model, firms spend 16% of total output on vacancy creation and keep 21% as profits. The piece rate does, however, imply a wage markdown relative to marginal productivity of 37%. As, direct empirical evidence on markdowns does not exist, they have to be inferred indirectly and the literature has yet to reach a consensus. While a markdown of 37% is large relative to the markdowns implied by estimates of rent sharing elasticities (Card et al., 2016), and at the upper end of markdowns implied by separation and quit elasticities Manning (2011), it is consistent with the estimates in (Yeh et al., 2022) whose production function approach yields wage markdowns of 35% in US manufacturing firms.

The piece rate is a key parameter affecting the employment effects of minimum wages as it determines by how much firms can increase wages until a given job becomes unprofitable. Put differently, given an initial distribution of wages, the same minimum wage hike will make more jobs unprofitable if markdowns the wage distribution are low. Very low markdowns are thus inconsistent with the observed absence of sizeable disemployment effects of past minimum wage hikes. In the robustness section, I analyze how the results change for lower wage markdowns, and show that an alternative markdown of only 15%, consistent with the rent sharing literature, would imply disemployment effects that are substantially larger than those documented by the empirical literature evaluating past reforms (e.g. Dustmann et al., 2022).

3.2 Estimation Strategy

The remaining parameters will be estimated using the simulated method of moments to match important aspects of the German labor market between 2011 and 2014.

Over half of these parameters are preference shifters for each employment level and demographic, $\gamma^j_{ft}, \gamma^j_{pt}, \gamma^j_{mj}$. These are primarily informed by the distribution of employment levels. I thus target the unemployment rate and the share of part-time and marginal jobs among all workers and within each demographic group.

The preference parameters γ_c and ϵ shape the relationship between workers' skill level and the surplus of working longer hours, and thus drive wage differences across job types. All else equal, high-skill workers have a higher surplus of working longer hours because of higher hourly wages. As γ_c increases, flow utility of consumption becomes more concave and the incentive to work full-time increases less in workers' skill level such that more high-skill (high-wage) workers work part-time. Similarly, the consumption-independent utility difference between full-time and part-time work scales more strongly with workers' skill level as ϵ increases. To the extent that workers prefer fewer hours, a higher ϵ leads to more high-skill workers in part-time jobs. I thus target selected quantiles of the distribution of wages conditional on employment level and the share of part-time and marginal jobs in different wage groups.

The curvature parameter ζ in the disutility of job search mainly affects the elasticity of job search with respect to the surplus of employment. Based on the quasi-experimental literature on the UI-elasticity of job finding probabilities, I target an average elasticity of 0.5 (e.g. Chetty, 2008; Schmieder et al., 2012).

The parameters ϕ_{su}, ϕ_{lu}, $\phi_{e_{ft}}$, $\phi_{e_{pt}}$, $\phi_{e_{mj}}$ determine the efficiency of job search in different labor market states. In addition to the unemployment rate, I target the average job finding probability out of short- and long-term unemployment as well as the probability of job-to-job transitions conditional on the current employment level.

The vacancy cost parameter κ_1 primarily affects overall labor market tightness by making vacancies more costly. In addition to the average job finding rates, I target the job vacancy rate. The curvature parameter κ_2 pins down the elasticity of vacancy posting with respect to the expected profitability of a match, $1/(1-\kappa_2)$. Since expected profits increase in firm productivity, κ_2 affects the firm size distribution. Increasing κ_2 will make it disproportionately more costly for more productive firms to grow large relative to less productive firms such that the standard deviation of log firm size decreases. I thus target the standard deviation of log firm size. To inform the mass of firms, m_f, I also target the mean of log firm size.

Finally, to guide the parameters of the skill and productivity distributions, μ_h^g, σ_h^g, α_p, θ_p, I use the fact that the model's wage equation is log-additive in worker skill and firm productivity. This allows me to directly compare the distribution of log human capital and log firm productivity to the empirical distribution of worker and firm fixed effects from an AKM model.[28] Specifically, I target selected quantile ratios of the distribution of worker (by gender) and firm fixed effects for full-time workers as well as selected quantile ratios of the distribution of full-time firm fixed effects weighted by the number of part-time and marginal jobs.

Note that targeting not only the distribution of wages, but also the distribution of worker and firm fixed effects ensures that the model also captures the degree of residual wage dispersion related to firm heterogeneity after controlling for worker heterogeneity.

Data The main data source is a 2% sample of administrative social security records of German workers (SIAB) from 2011 to 2014. The SIAB is a linked employer-employee data set containing information on daily earnings and employment levels (full-time, part-time and mini-job) for all German employees that pay social security contributions.[29] Sociodemographic characteristics (apart from gender and age) are only available for nonemployed workers. I thus complement the SIAB sample with survey data from the German Socioeconomic Panel (SOEP) which contains annual information on more than 15 thousand workers. For firm-level moments I use

[28]As I only have access to a 2% sample of the linked employer-employee data, I estimate the empirical distribution of worker and firm-class fixed effects using a clustered AKM approach (Bonhomme et al., 2019). I first cluster firms based on their wage distributions and use firm-class fixed effects instead of firm fixed effects. In order to reduce the impact of measurement error due to the lack of precise hourly wage information, I estimate the empirical model using full-time workers. See AppendixC for details.

[29]The data does not cover civil servants as they do not pay social security contributions.

administrative data from the Establishment History Panel and the Job Vacancy Survey of the Institute for Employment Research (IAB) at the German Federal Employment Agency. I focus on workers aged 25 to 60. See Appendix C for more details on the data.

Estimation Procedure I estimate the model using a two-step multiple-restart procedure similar to the TikTak-estimation method proposed by Arnoud et al. (2019) and used by Guvenen et al. (2020). In the first stage, I search a compact parameter space by evaluating the objective function at 1.12 million quasi-random Sobol points. I then select the best 560 points as starting points for local minimizations and pick the local minimizer with the lowest local minimum as the global minimizer.

The objective function to be minimized is the weighted sum of squared arc-percent differences between the model-implied moments and their empirical counterparts.

$$F(\theta) = \sum_{k=1}^{K} \omega_k \left(\frac{m_k(\theta) - d_k}{\frac{1}{2}\left(|m_k(\theta)| + |d_k|\right) + \psi_k} \right)^2 \tag{12}$$

Here, θ is a vector of internally estimated parameters, $m_k(\theta)$ is the model-implied moment, d_k is the empirical moment, ω_k is the weight, and ψ_k is a small constant to avoid division by zero. I provide more details on the moments and their targets in Appendix C.

Identification Checks To complement the intuitive discussion of the identification strategy, I document how important moments—the median full-time wage, the UI-elasticity of the job finding rate, the share of part-time and marginal jobs, the job finding rate out of unemployment, the standard deviation of log firm size, and the share of the variance of log wages explained by firm heterogeneity—change when important parameters are varied around their baseline value.[30] These moment-parameter relationships are largely in line with the intuition provided above.

Unsurprisingly, the median and standard deviation of log wages are primarily driven by the skill distribution parameters μ_h^g and σ_h^g (Figures D.1 and D.2). The UI-elasticity of the job finding rate varies significantly with parameters that increase the surplus of employment, μ_h, α_p, θ_p, and r, but is mainly driven by the convexity of search disutility, ζ (Figure D.5). The share of part-time and marginal jobs is mainly driven by the preference shifters γ_x, which in turn do not have a major influence on other moments (Figure D.7 and D.8). The share of marginal jobs additionally increases significantly in the efficiency of search effort as this makes it easier for workers to find their preferred employment level, and decreases in parameters that increase hourly wages (μ_h, r) as higher hourly wages make it more attractive to work longer hours. The job finding rate out of unemployment is positively affected by the search efficiency ϕ^{su}, mean log worker productivity μ_h, and negatively by the convexity of search disutility, ζ, the concavity of consumption utility, γ_c, which reduces the surplus of employment, and by the vacancy-elasticity of matches, ξ, which reduces the importance of aggregate search in the matching function (Figure D.9). Finally, both the standard deviation of log firm size and the

[30] See Appendix D for more details.

Table 1: Worker Parameters

Name	Description	Value	Source
Skill Distribution of Men			
μ	Mean of log(h)	2.568	sim. method of moments
σ	Std. dev. of log(h)	0.510	sim. method of moments
Skill Distribution of Women			
μ	Mean of log(h)	2.548	sim. method of moments
σ	Std. dev. of log(h)	0.548	sim. method of moments
Men, Single			
γ_{ft}	State utility of $s = ft$	-0.187	sim. method of moments
γ_{pt}	State utility of $s = pt$	-0.167	sim. method of moments
γ_{mj}	State utility of $s = mj$	0.171	sim. method of moments
Men, Married			
γ_{ft}	State utility of $s = ft$	0.520	sim. method of moments
γ_{pt}	State utility of $s = pt$	-0.024	sim. method of moments
γ_{mj}	State utility of $s = mj$	0.307	sim. method of moments
Women, Single, No Kids			
γ_{ft}	State utility of $s = ft$	-0.125	sim. method of moments
γ_{pt}	State utility of $s = pt$	0.320	sim. method of moments
γ_{mj}	State utility of $s = mj$	0.781	sim. method of moments
Women, Single, Kids			
γ_{ft}	State utility of $s = ft$	-0.861	sim. method of moments
γ_{pt}	State utility of $s = pt$	0.360	sim. method of moments
γ_{mj}	State utility of $s = mj$	0.905	sim. method of moments
Women, Married			
γ_{ft}	State utility of $s = ft$	-0.174	sim. method of moments
γ_{pt}	State utility of $s = pt$	0.854	sim. method of moments
γ_{mj}	State utility of $s = mj$	1.863	sim. method of moments
All Workers			
β	Discount factor	0.985	–
γ_c	CRRA parameter	0.923	sim. method of moments
ζ	Search disutility (convexity)	2.107	sim. method of moments
ϵ	Relation btw. h and state utilities	-0.118	sim. method of moments

share of the variance of log wages attributable to firm heterogeneity, are significantly affected by the convexity of the vacancy posting cost function, κ_2, and the concavity of consumption utility, γ_c (Figure D.10 and D.11).

3.3 Parameter Estimates and Model Fit

Tables 1 and 2 show the pre-set and estimated parameters of the model.

Employment States and Flows Table 1 shows that, apart from married men, all workers prefer to work fewer hours, as $\gamma_{ft} < \gamma_{pt} < \gamma_{mj}$. These preference shifters primarily affect the share of part-time and marginal jobs, which the model matches well (Figure 1). In the model, 25.3% of workers work part-time compared to 24.0% in the data, and 9.2% work marginal jobs compared to 9.6% in the data. Driven by heterogeneity in γ_{ft}, γ_{pt} and γ_{mj} across demographic

Table 2: Firm, Labor Market and Policy Parameters

Name	Description	Value	Source
Firms			
β_f	Discount factor	0.985	–
m	Mass of firms	0.050	sim. method of moments
α_p	Scale of $log(p)$	2.916	sim. method of moments
θ_p	Shape of $log(p)$	0.095	sim. method of moments
κ_1	Vacancy posting cost (weight)	145.6	sim. method of moments
κ_2	Vacancy posting cost (convexity)	1.379	sim. method of moments
Labor Market			
ξ	Vacancy-elasticity of matches	0.3	Petrongolo & Pissarides (2001)
r	Wage piece-rate	0.630	agg. labor share, ILO
e_{ft}	Hours worked in full-time jobs	1.0	normalized
e_{pt}	Hours worked in part-time jobs	0.615	Dustmann et al. (2022)
e_{mj}	Hours worked in marginal jobs	0.223	Dustmann et al. (2022)
$\pi_{su\|e_{ft}}$	Transition from e_{ft} to su	0.010	SIAB
$\pi_{su\|e_{pt}}$	Transition from e_{pt} to su	0.014	SIAB
$\pi_{su\|e_{mj}}$	Transition from e_{mj} to su	0.023	SIAB
ψ_{ft}	Godfather shock, $x = ft$	0.017	SIAB
ψ_{pt}	Godfather shock, $x = pt$	0.022	SIAB
ψ_{mj}	Godfather shock, $x = mj$	0.064	SIAB
$\pi_{lu\|su}$	Transition from su to lu	0.084	sim. method of moments
ϕ_{su}	Search efficiency, $s = su$	0.475	sim. method of moments
ϕ_{lu}/ϕ_{su}	Relative search efficiency, $s = lu$	0.378	sim. method of moments
ϕ_{ft}/ϕ_{su}	Relative search efficiency, $s = e_{ft}$	1.179	sim. method of moments
ϕ_{pt}/ϕ_{su}	Relative search efficiency, $s = e_{pt}$	0.992	sim. method of moments
ϕ_{mj}/ϕ_{su}	Relative search efficiency, $s = e_{mj}$	1.057	sim. method of moments

groups, the model matches the fact that women, specifically single women with kids and married women, are substantially less likely to work full-time. The model also closely matches the overall unemployment rate of 6.4% and the share of long-term unemployment (51.8%), but somewhat underestimates heterogeneity in unemployment rates across demographic groups.

The estimated convexity of search disutility is slightly larger than two, $\zeta = 2.107$, and generates an average UI-elasticity of the job finding rate of -0.495, almost perfectly matching its target of -0.5. Regarding labor market flows, the model matches the substantial difference in job finding rates between short- and long-term unemployed workers observed in the data (29.6% vs. 6.7%) by making search effort less effective for long-term unemployed workers, $\phi_{lu}/\phi_{su} = 0.378$. In contrast, on the-job-search is estimated to be similarly effective relative to short-term unemployment, which gives a probability of job-to-job transitions of 4.4% in the model compared to 3.5% in the data. For the transition analyses in Section 5.3, it is important to keep in mind that labor market turnover in Germany and other European countries is substantially lower compared to the US or other anglosaxon countries (Hobijn and Şahin, 2009).[31] In Section 5.4, I will show that frictional unemployment during the transition between two steady states plays less of a role in a version of the model with higher job finding rates.

Wages For the analysis of long-run effects of minimum wages, it is important that the model matches how many and what kind of jobs are affected by a given minimum wage. Panel A of Figure 2 shows that the model matches the distribution of wages well. Overall, the standard deviation of log wages in the model is 0.507 compared to 0.512 in the data. In particular, 11.1% (11.4% in the data) of workers earn less than 8.5€ the initial minimum wage in Germany, 19.7% (18.7%) earn between 8.5€ and 12.5€ and 31.9% (34.7%) earn between 12.5€ and 20€.

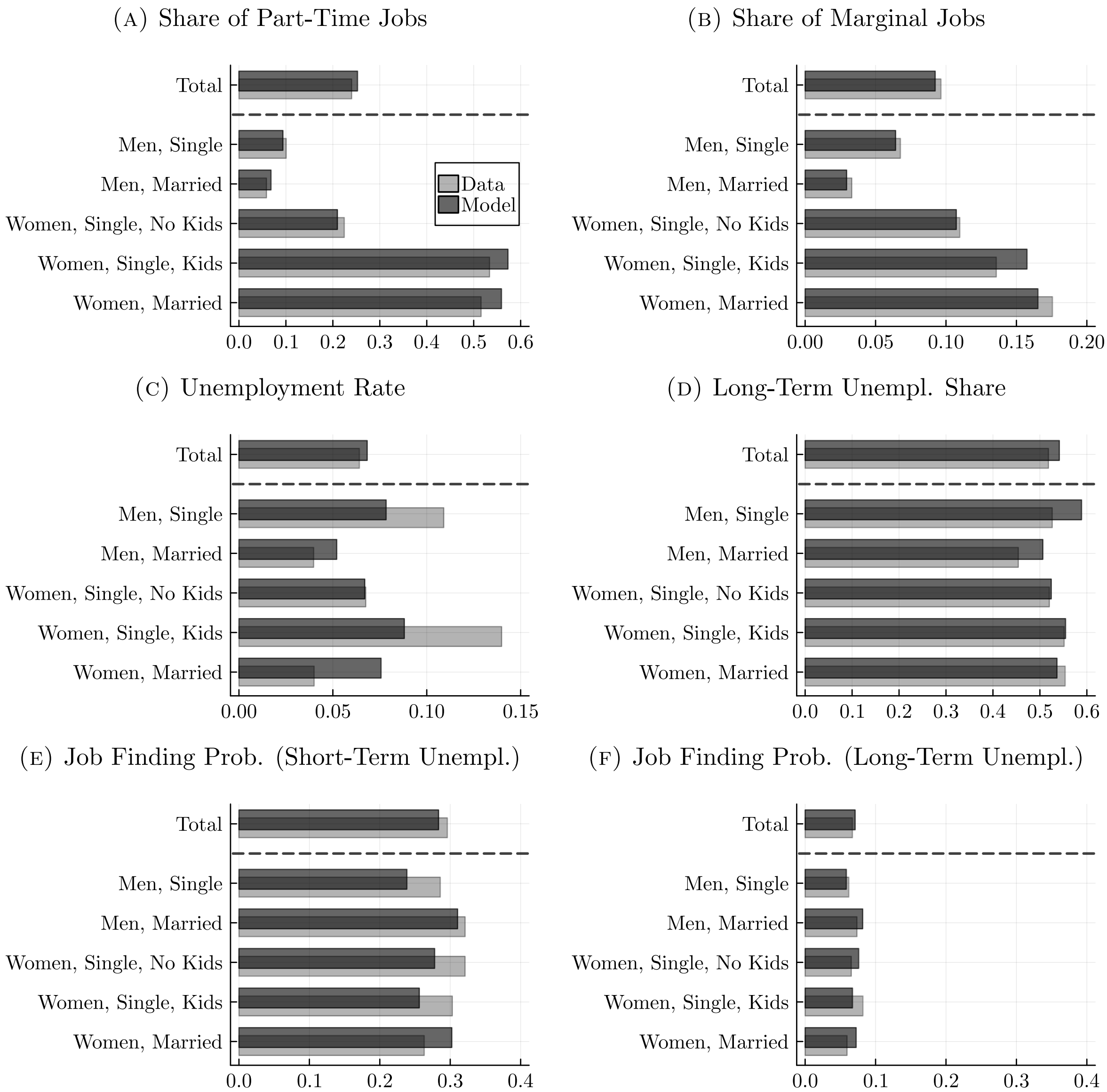
While the model underestimates the gender gap in wages[32], the model provides a good approximation to the conditional wage distributions of full-time, part-time and marginal workers (Panels B and C of Figure 2). Both in the model and in the data, the wage distribution in full-time jobs first order stochastically dominates that of part-time jobs, which in turn dominates that of marginal jobs. The median full-time worker earns 18.9€ (18.4€) compared to 14.8€ (15.3€) for part-time workers and 8.7€ (8.2€) for marginal workers. Hence, the model captures the fact that a disproportionate share of minimum wage workers are in part-time and marginal jobs.[33]

[31] The quarterly job destruction rates, estimated directly from the data, are 1.0%, 1.9% and 3.0% for full-time, part-time and marginal jobs, respectively.

[32] In the model, 34.6% of women earn less than 12.5€ compared to 39.2% in the data.

[33] Appendix Figure A.2 shows the distribution of job types and gender within selected wage groups.

FIGURE 1: Model Fit: Labor Market States and Job Finding Probabilities



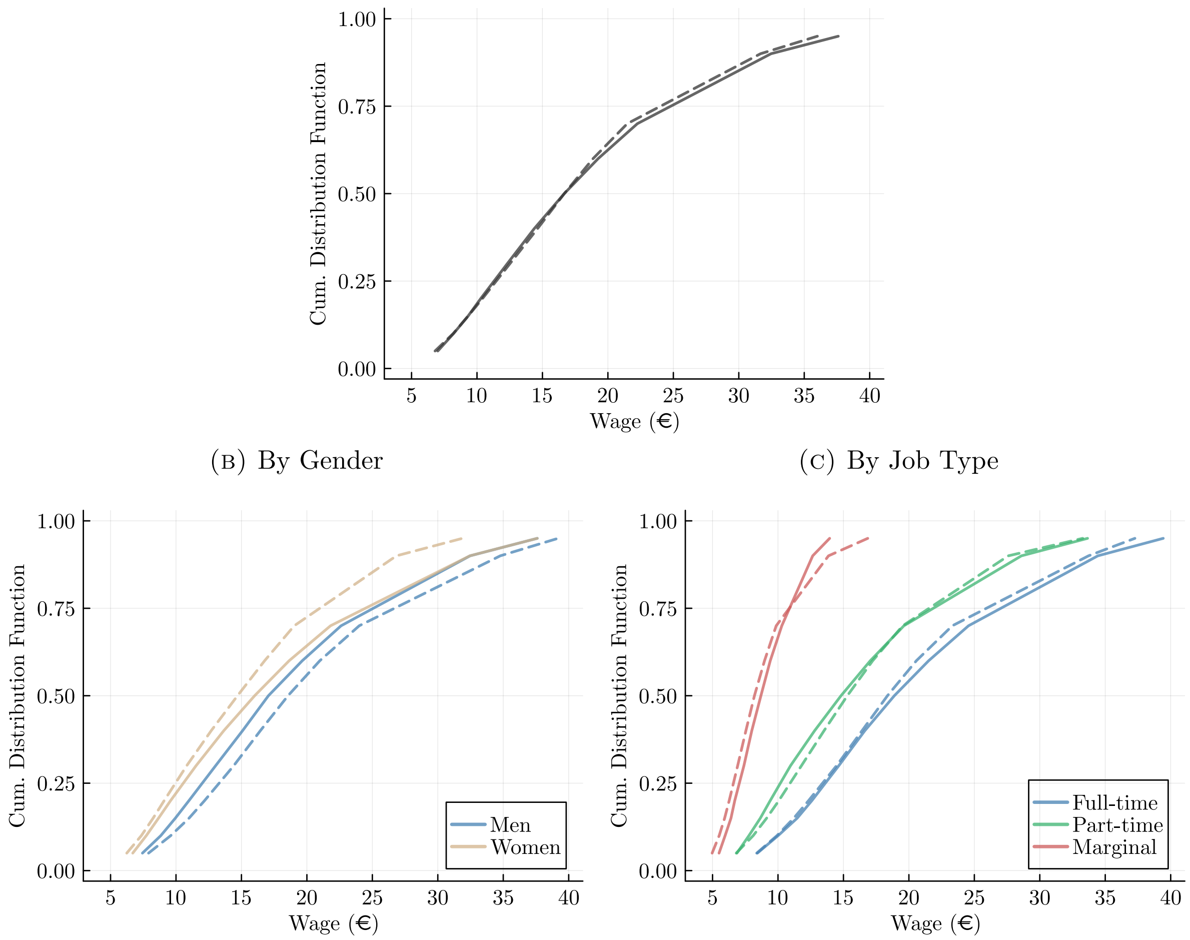
Notes: This figure shows the distribution of labor market states and job finding probabilities in the model (dark bars lines) and the data (light bars) by demographic group. Panels A and B show the share of part-time and marginal jobs among all jobs, Panel C shows the unemployment rate, Panel D shows the share of long-term unemployment among all unemployed, and Panels E and F show the average job finding probabilities for short-term and long-term unemployed workers.

Figure 2: Model Fit: Wage Distribution

(a) All Workers

(b) By Gender

(c) By Job Type



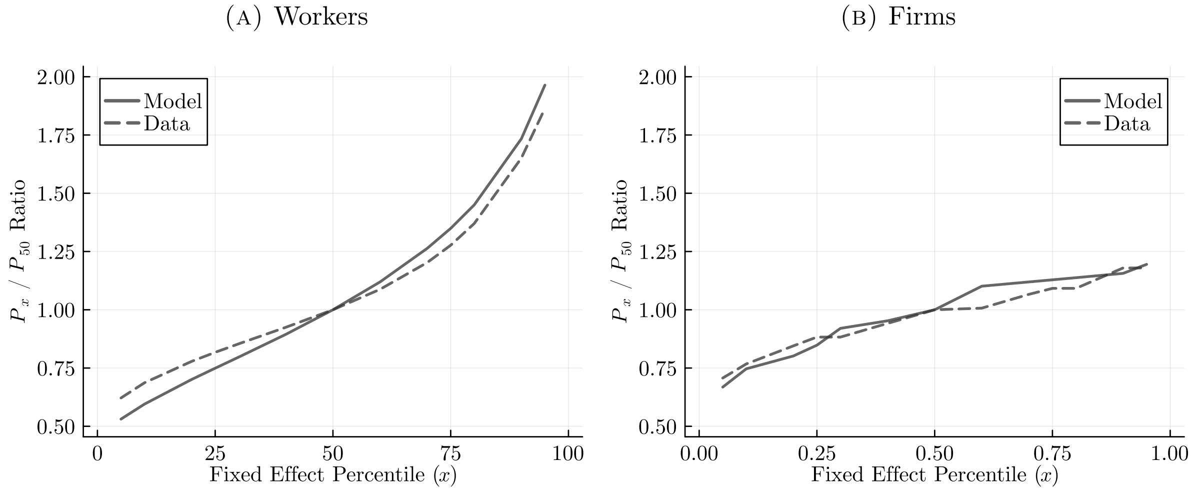
Notes: This figure shows the cumulative distribution function of wages in the model (solid lines) and the data (dashed lines) for different groups of workers. The top panel shows the overall wage distribution. The bottom left panel shows the wage distribution by gender and the bottom right panel shows the wage distribution by job types (full-time, part-time, marginal).

Worker and Firm Fixed Effects As afore-mentioned, the model should also be able to match the distribution of worker and firm fixed effects to ensure that residual wage dispersion, which creates room for reallocation towards high-productivity firms, is not implausibly high. Panel A of Figure 3 shows the dispersion in worker and firm fixed effects in the model and the data. The model matches the distribution of worker and firm fixed effects reasonably well.

As in the data, the variation in worker fixed effects is much larger than the variation in firm fixed effects. While the standard deviation of worker fixed effects equals 0.395, the standard deviation of firm fixed effects is only 0.125. Hence, worker heterogeneity accounts for 73.0% of the total variance in log wages, compared to only 7.4% for firm heterogeneity and 19.6% for worker-firm sorting. If anything, the model underestimates the share of the variance explained by firm heterogeneity, which is 12.8% in the data when using the clustered AKM approach.[34]

[34] The variance decomposition reported in Card et al. (2013) who use the standard AKM approach on the universe of German data for the years 2002 to 2009, suggests that 21% of the variance is due to firm heterogeneity.

Figure 3: Model Fit: Worker and Firm Fixed Effect Distributions



Notes: This figure shows the ratio between different percentiles and the median of the distributions of the worker and firm fixed effects in the model (solid lines) and the data (dashed lines). The left panel shows the distribution of the worker fixed effects. The right panel shows the distribution of the firm fixed effects.

Appendix Figure A.3 shows that the mean-min wage ratio conditional on worker skill in the model—another measure of residual wage dispersion proposed by Hornstein et al. (2011)—ranges between 1.5 in the lowest and 2.0 in the highest worker skill decile. Given the presence of on-the-job search and long-term unemployment with skill-independent transfers, these values are in line with the findings in Hornstein et al. (2011).[35]

Appendix Figure A.4 shows that the model also does a good job of capturing the dispersion in firm fixed effects conditional on employment level. However, the model overestimates average productivity differences between part-time or marginal jobs on the one hand, and full-time jobs on the other hand. That is, relative to the data, a disproportionately large fraction of marginal jobs are offered by low-productivity firms.

Firm Size Distribution The vacancy posting cost function is estimated to be $\kappa_2 = 1.38$, similar to the estimate of 1.46 in Engbom and Moser (2022). The rather modest degree of convexity is required for the model to approximate the empirical standard deviation of log firm size of 2.19. The estimated model delivers a value of 2.02 and this value would decrease further if vacancy posting costs were more convex (see Appendix Figure D.10). The relatively low value for κ_2 implies that firms' vacancy posting is relatively elastic to changes in the profitability of working relationships and hence minimum wages. That is, for a one percent reduction in expected match surplus, firms will reduce their vacancy posting by $1/(\kappa_2 - 1) = 2.6\%$.

However, recent work by Bonhomme et al. (2023) shows that the variance share of firm effects roughly halves and the variance share of worker-firm sorting increases when correcting for limited mobility bias. Taking this into account, the model's firm variance share of 7.4% is in line with the data.

[35] Engbom and Moser (2022) report mean-min wage ratios between 1.3 and 3.0 for their model of the Brazilian labor market.

4 The German Minimum Wage Reform of 2015

In 2015, the German government introduced a federal minimum wage of 8.5€ (Kaitz index of 47%) that cut deep into the wage distribution, affecting more than 10% of all jobs. In this section, I use the estimated model to analyze how this large reform affected employment, output, and worker welfare. I will also show that the model's predictions on employment and reallocation effects are consistent with empirical analyses of this reform (Dustmann et al., 2022).

4.1 Aggregate Effects

Table 3 compares the steady state without a minimum wage (column 1) to the steady state with a minimum wage of 8.5€ (column 2). The difference between the new and the old stationary equilibrium is shown in column 3.

Relative to the steady state without a minimum wage, the employment rate stays virtually unchanged in the steady state with a minimum wage of 8.5€Ḣowever, the negligible change in the number of jobs masks heterogeneity across employment levels. In particular, while the share of marginal jobs among all jobs drops from 9.2% to 7.5%, the share of part-time and full-time jobs increases by 1.1 and 0.7 percentage points respectively. In addition, labor market turnover decreases slightly. On the one hand, the average job finding rate out of unemployment, $\Pr(e|u)$, falls by 0.28 percentage points (1.7%). On the other hand, the average job destruction probability falls by 1.5% because of reallocation away from relatively unstable marginal jobs toward more stable part-time and full-time jobs.

Average wages in the new stationary equilibrium are up by 1.26%. Part of this increase is driven by reallocation toward more productive firms, as average firm productivity rises by 1.75%. In other words, workers now work at firms where they would have received higher wages even in the absence of a minimum wage. Average gross earnings increase by more than wages (1.39%), reflecting the shift toward jobs with longer hours (1.12%). Taxes and transfers result in a 0.72% increase in average incomes and thus consumption. The relatively weak increase in incomes follows from the fact that many low-skill workers top up their earnings with subsistence benefits. As earnings increase, these transfers are reduced.

In the absence of significant disemployment effects, the increase in average firm productivity and hours worked leads total output to grow by 1.12%. The labor share in total output increases only slightly by 0.16 percentage points. For the government, the increase in labor earnings leads to a 1.44% increase in tax revenues and an 7.93% decrease in transfer payments such that the government budget increases by roughly 2.6%.

Finally, the reform increased worker welfare by 0.4% in consumption equivalents. To see why the overall welfare gain is relatively small, recall that workers' utility is affected by consumption, search effort and hours worked. As these components enter the flow utility function additively, workers' lifetime utility can also be split into three parts, one for each component. To understand how the overall welfare effect aggregates gains and losses from changes in consumption, search and hours worked, I compute the consumption equivalent welfare change that would materialize

Table 3: Long-Run Effects of the German Minimum Wage Introduction

	(1)	(2)	(3)
	Baseline	New Equilibrium	
	Value	Value	Change
Labor Market States			
Employment Rate	0.9318	0.9316	-0.01%
Full-Time Share	0.6564	0.6627	0.96%
Part-Time Share	0.2515	0.2628	4.47%
Marginal Share	0.0920	0.0745	-19.05%
Long-Term Unempl. Rate	0.0370	0.0373	0.91%
Transition Probabilities			
Job Finding Prob.	0.1683	0.1655	-1.70%
Job Destruction Prob.	0.0123	0.0121	-1.52%
Mean Log Wage	2.801	2.831	1.05%
Mean Log Productivity	0.631	0.651	3.26%
Mean Log Hours	3.401	3.422	0.61%
Mean Log Earnings	6.203	6.253	0.81%
Wages, Earnings & Incomes			
Mean Wage (€)	18.66	18.90	1.26%
Mean Productivity	1.906	1.940	1.75%
Mean Hours	32.42	32.79	1.12%
Mean Earnings (€)	2,768	2,807	1.39%
Mean Income (€)	2,129	2,144	0.72%
Macro Aggregates			
Output per capita (€)	4,097	4,143	1.12%
Hours per capita	30.21	30.54	1.11%
Labor Share	0.630	0.632	0.26%
Transfers per capita (€)	94	86	-7.93%
Taxes per capita (€)	882	895	1.44%
CE Welfare Effects			
Total Effect			0.40%
Consumption Component			1.09%
Hours Component			-0.62%
Search Component			0.14%

Notes: This table shows the long-run effects of the German minimum wage introduction in 2015. Columns 1 and 2 show the pre- and post-reform moments and columns 3 shows the percentage difference between the two. All Euro values are in 2014 prices.

TABLE 4: Heterogeneous Effects of the German Minimum Wage Introduction

	Men		Women		
	Single	Married	Single		Married
			No Kids	Kids	
Wages and Incomes					
Share with Minimum Wage	5.29	5.28	7.98	9.72	9.14
Δ Mean Wage (%)	0.99	0.91	1.49	1.79	1.67
Δ Mean Earnings (%)	1.18	0.94	1.59	2.08	2.01
Δ Mean Income (%)	0.43	0.68	0.43	0.36	1.17
Labor Market States					
Δ Full-Time (p.p.)	1.12	0.27	0.88	0.28	0.35
Δ Part-Time (p.p.)	0.26	0.24	1.27	1.98	2.26
Δ Marginal (p.p.)	-1.29	-0.48	-2.21	-2.47	-2.71
Δ Unemployed (p.p.)	-0.09	-0.04	0.06	0.20	0.09
Welfare					
Δ Total Effect (%)	0.40	1.22	-0.13	-0.54	-0.03
Δ Consumption Comp. (%)	0.69	1.24	0.59	0.40	1.65
Δ Hours Comp. (%)	-0.12	0.22	-0.81	-1.10	-1.76
Δ Search Comp. (%)	0.09	0.22	0.07	0.04	0.15

Notes: This table shows the long-run effects of the German minimum wage introduction in 2015 separately for each of the five demographic groups. Welfare effects are expressed in consumption equivalents. The component-specific welfare effects are calculated as the consumption equivalent welfare change needed to match the change in lifetime utility when only one component of the utility function is changed. Total lifetime utility is the sum of the discounted sums of flow utilities from consumption, hours worked and search effort. Nevertheless, when expressed in terms of consumption equivalents, the CE welfare effects for each component do not sum to the total CE welfare effect. See Appendix B for details on how overall and component-specific welfare changes are computed.

if only one of these components is changed to the new equilibrium. If lifetime disutility of hours and search is fixed, the minimum wage reform would imply an average consumption equivalent welfare gain of 1.1% due to increased earnings and consumption. In contrast, if consumption-related utility is fixed and only the change in the disutility of hours worked is taken into account, the minimum wage leads to a 0.62% decrease in welfare. The change in the disutility of search is quantitatively small.[36] Hence, the increase in consumption utility is partly offset by an increase in disutility from working longer hours.

In sum, the introduction of the minimum wage moves the economy into an equilibrium with the same number of jobs but an improved composition of jobs, thereby increasing output increases. The minimum wage thus partly removes misallocation toward low-earnings jobs created by search frictions and transfers to workers with low-earnings jobs. While output and the government balance expands, reduced transfers due to higher earnings and higher disutility from working longer hours significantly dampen the increase in consumption and worker welfare.

[36] Note also that the three components do not sum to the total welfare effect because of the non-linearities in the components of the utility function.

4.2 Heterogeneity by Demographics

The aggregate effects mask interesting heterogeneity across demographic groups, which is documented in Table 4. First, women are significantly more likely to receive the new minimum wage of 8.5€ per hour. Consequently, women experience larger increases in average wages and earnings.

However, the different demographic groups differ in how the increase in earnings translates into increases in disposable income. The link is weekest for single women with kids who, before the reform, rely relatively more on government transfers to top up their earnings. For them, the increase in earnings is largely offset by the phase-out of transfers. In particular, the 0.36% increase in average incomes is less than a fifth of the 2.08% increase in earnings. In addition, Married women, who have a similar share of minimum wage workers but do not receive any transfer payments because of spousal income and joint taxation, experience a much stronger increase in average incomes (1.17%) compared to single women with kids.

The middle panel of Table 4 displays the percentage point changes in the distribution of labor market states. While the reallocation pattern away from marginal toward part-time and full-time jobs is the same qualitatively, there is substantial variation in magnitude. Men and single women without kids move to both part-time and full-time jobs. In contrast, married women and single women with kids mainly switch from marginal to part-time jobs due to their high disutility of working full-time.

The differences in income hours effects translate into heterogeneous welfare effects. While all groups of workers experience an increase in income and hence consumption, only men experience a positive overall welfare effect. Married men benefit most with a consumption equivalent welfare increase of on average 1.22%.[37] Women, and in particular single women with kids, experience a welfare loss as the disutility from working longer hours is equivalent to a decrease in consumption between 0.81% and 1.76%. Hence, wage gains of 1.5% or more can be turned into welfare losses if reallocation toward jobs with longer hours are sufficiently high.

4.3 Transitional Dynamics

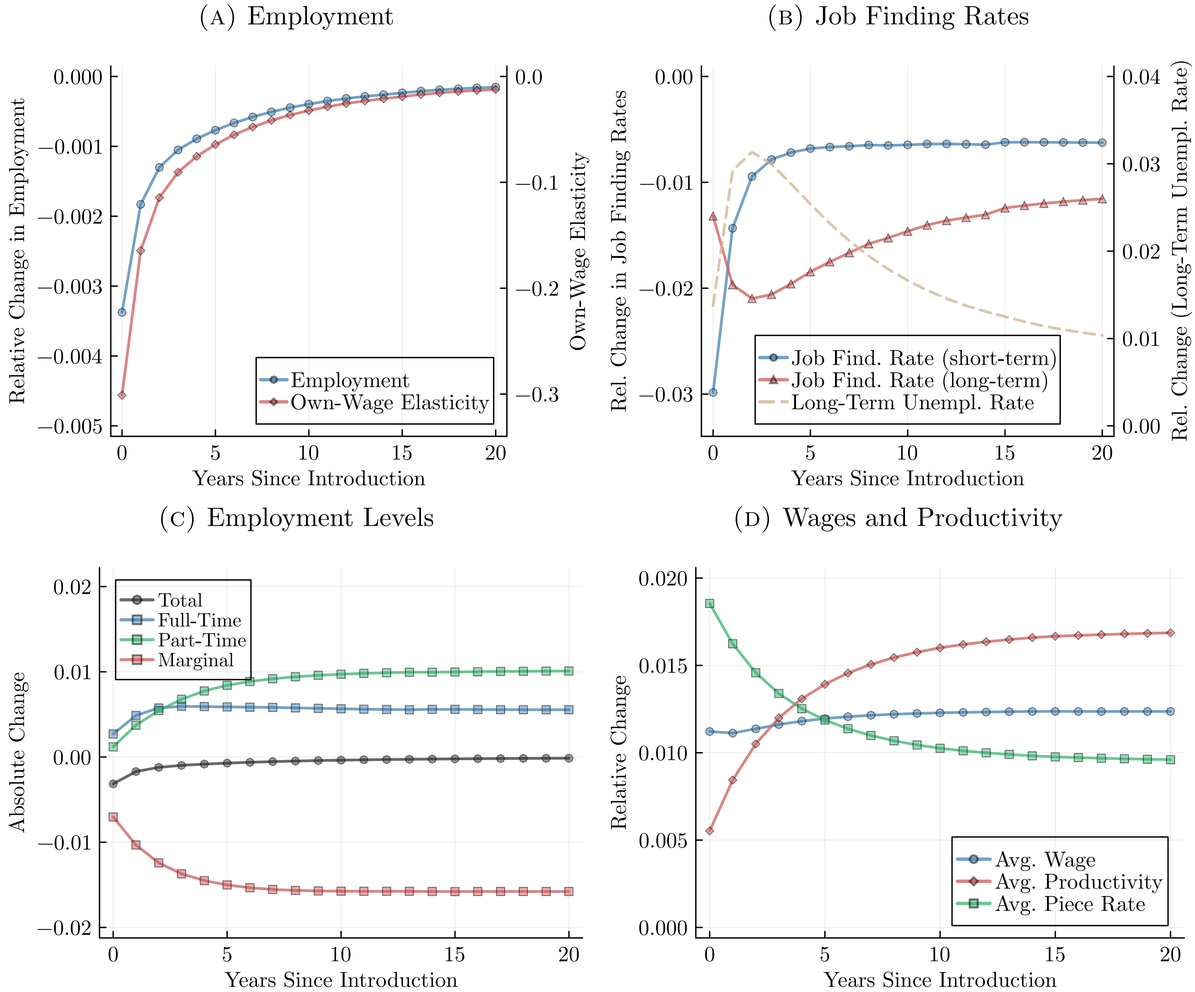
In the presence of search frictions, the process of worker reallocation takes time. Workers whose jobs survive the introduction of the minimum wage will gradually transition to more productive firms or jobs with longer hours. More importantly, the minimum wage will make some jobs unprofitable. Thus, some workers become unemployed and finding a new job takes time. Hence, it is important to study the dynamic effects of the minimum wage reform.[38]

Figure 4 shows how the economy reacts to the reform. Time $t = 0$ corresponds to the quarter in which the minimum wage is introduced. Panel A shows that the initial drop in total employment is indeed 0.3 percentage points larger than the long-run employment effect. While the employment effect is small both in the short- and long-run, convergence to the new steady

[37] Compared to single men, married men are less affected by the phase-out of transfers and have a lower tax rate because of joint taxation.

[38] Appendix E explains how the transition path is computed.

Figure 4: Dynamic Reallocation Effects of the German Minimum Wage Introduction



Notes: This figure shows the dynamic effects of the German minimum wage introduction. Panel A shows the relative change in overall employment (left axis) and the own-wage elasticity (right axis). Panel B shows the relative change in the job finding rates of short- and long-term unemployed workers (left axis) and the share of long-term unemployment (right axis). Panel C shows the absolute change in the share of employed workers and the share of workers with full-time, part-time and marginal jobs. Panel D shows the relative change in average wages, firm productivity, and the average piece-rate.

state takes years rather than months. This is mainly due to the fact that labor market turnover in Germany is low. While the own-wage elasticity of employment, i.e., the relative change in employment following a 1% increase in wages, is -0.3 at impact it quickly decreases to -0.16 one year and -0.06 five years after the reform. Panel B shows that this sluggish convergence is driven by an influx of (low-skill) workers into long-term unemployment (right axis), where job finding rates are low even before the minimum wage and decrease further after the reform as firms post fewer vacancies in low-skill segments of the labor market where the minimum wage bites most.

Panels C and D show that reallocation away from marginal jobs and low-productivity firms also takes time. In particular, Panel D shows that while average wages jump up immediately and don't exhibit much of a transition, the gradual reallocation toward more productive firms goes hand in hand with a recovery of average profit margins. That is, higher wages are increasingly the result of working for more productive firms rather than increases in piece-rates.

4.4 Comparison with Reduced-Form Evidence

While the analysis of aggregate and welfare effects of the German minimum wage introduction are a valuable addition to the empirical literature on the reform, the latter may be even more important as a validation exercise for the model. Given that the model was not estimated to match observed minimum wage effects, it is ex-ante unclear whether the model delivers plausible predictions on minimum wage effects.

Hence, I now briefly discuss how the model's predictions line up with the available reduced-form evidence on the initial introduction of the minimum wage which can be seen as an independent test of the model. There are several studies documenting the short-run effects of the 2015 minimum wage reform using individual or regional variation in the bite of the minimum wage (e.g. Garloff, 2016; Caliendo et al., 2017; Holtemöller and Pohle, 2017; Burauel et al., 2020; Dustmann et al., 2022). The empirical findings of Dustmann et al. (2022), henceforth DLSUB, boil down to the following points.

First, hourly wages increased significantly and in line with near full compliance from 2014 to 2016. According to DLSUB, average wages grew by 1.16% to 1.36%, compared to 1.26% in the model.[39]

Second, and again consistent with the model, overall employment effects are very small or even slightly positive.[40] However, the minimum wage induced a shift from marginal jobs toward part-time and full-time jobs. For example, DLSUB find that employment in terms of full-time equivalents of minimum wage workers, i.e., worker earning less than the minimum wage before the reform, increased by 2.5 percentage points, whereas the unweighted employment effect is close to zero. In the model, employment in full-time equivalents increases by 2.4 percentage points. As the overall employment of minimum wage workers two years after the reform decreases

[39]The numbers from DLSUB refer to their region-level estimates in Table 7 (multiplied with the average GAP measure which is the increase in average wages under full compliance with the minimum wage).

[40]In addition and consistent with the model, turnover rates decreased as both job finding and separation rates were reduced (Bossler and Gerner, 2016).

by 1.3 percentage points, the difference between the unweighted employment effects and the full-time equivalent effects is somewhat larger in the model than in the data. [41]

Third, there is also evidence that the minimum wage reallocated workers to larger, more productive firms. DLSUB attribute 17% of the increase in daily wages of minimum wage workers between 2014 to 2016 to reallocation toward more productive firms. In the model, 20.0% comes from reallocation. Similarly, the diff-in-diff estimates in DLSUB imply that the average firm fixed effect increased by up to 0.92% in the data. Over the same time period, the corresponding number in the model is 0.84%.[42] Hence, while the model predicts somewhat stronger reallocation toward high-productivity firms, the qualitative pattern is the same and the quantitative difference is small.

Finally, as in the model, the empirical evidence indicates that reallocation effects take time to materialize. While wages jump up immediately after the reform, different estimates of reallocation increase substantially in both post-reform years analyzed by DLSUB. For example, the cumulative increase in the average firm fixed effect after two years is roughly twice as high as the increase after one year.[43] In the model, the corresponding cumulative increase is 69.0% higher in year two relative to year one. In addition, as in the model, the initial employment effects are small, but more positive in year two than in year one.

In sum, the estimated model captures all these effects qualitatively and does a good job of replicating them quantitatively. The fact that the model not only matches well the labor market moments in the pre-reform period, but is also broadly consistent with the rich reduced-form evidence on the minimum wage reform builds credibility for the following analysis of higher minimum wages.

5 Counterfactually High Minimum Wages

In this section, I use the model to analyze how counterfactually high minimum wage levels impact employment, output and worker welfare. First, I study the long-run effects on employment and reallocation (Section 5.1), as well as heterogeneous welfare effects (Section 5.2). Second, I analyze the transition dynamics and compare short- and long-run effects (Section 5.3). Finally, I discuss the robustness of the results with respect to important parameters (Section 5.4).

[41]The reduced-form estimates are based on Table 2, column 1 in DLSUB. Another way of assessing the role of hours changes is to compare earnings and wage effects. In the data, earnings of minimum wage workers who remain employed grow 65% more strongly than their hourly wages. In the model, the analogously computed earnings effect is 45.0% larger than the wage effect. In Appendix Figure A.5, I show that the share of marginal jobs in Germany declined following the introduction of the minimum wage. While there is no clean control group, the deviation from trend between 2015 and 2019 is consistent with the model's prediction.

[42]The empirical increase in the average firme fixed effect is calculated as (0.468×0.017) where 0.468 is the diff-in-diff estimate (Table 7), and 0.017 is the nation-wide GAP measure. In the model, I analogously average the increase in mean log firm productivity across the first two post-reform years.

[43]See Figure 8, Panel D in DLSUB. Other measures of reallocation also increase substantially from year one to year two (Figures 2 to 5).

5.1 Long-Run Employment and Reallocation Effects

I first take a long-run perspective by comparing the stationary equilibrium that emerges for different minimum wages to the baseline equilibrium. Throughout the analysis, I plot the minimum wage's Kaitz index, i.e., the ratio of the minimum wage to the baseline full-time median wage, rather than the nominal minimum wage itself to facilitate comparisons across different economies and time periods.[44]

Figure 5 shows the effects on steady-state employment, total hours worked, average productivity, and total output. Panel A shows that total employment remains virtually unchanged for moderate minimum wages and does not drop below the baseline level for minimum wages below 70%. However, for even higher minimum wages, employment starts to decline rapidly. For example, a Kaitz index of 80% reduces employment by over 2% compared to the baseline without a minimum wage, corresponding to an increase in the unemployment rate by over 30%.

Panels B and C capture the reallocation effects that change the composition of jobs. Panel B shows that the minimum wage not only affects the extensive margin of employment but also hours worked. While the number of employed workers is essentially flat for moderate minimum wages, total hours worked increase significantly until a Kaitz index of 75%. The increase in hours worked is driven by the fact that higher (hourly) wages incentivize workers to take on more hours. For example, at a Kaitz index of 65%, i.e., the highest value without aggregate job loss, the minimum wage increases the average earnings difference between full-time and marginal jobs by 36.0%. Holding labor market tightness and firms' vacancy posting constant, this would increase search effort among marginal workers by 6.0%, whereas search effort of full-time workers decreases by 14.0%. This change in on-the-job search behavior decreases (increases) the expected match duration of marginal (full-time) jobs and hence lowers firms' valuation of vacancies for marginal jobs relative to full-time jobs. In equilibrium, a Kaitz index of 65% leads to an increase in hours worked of 4.9%.

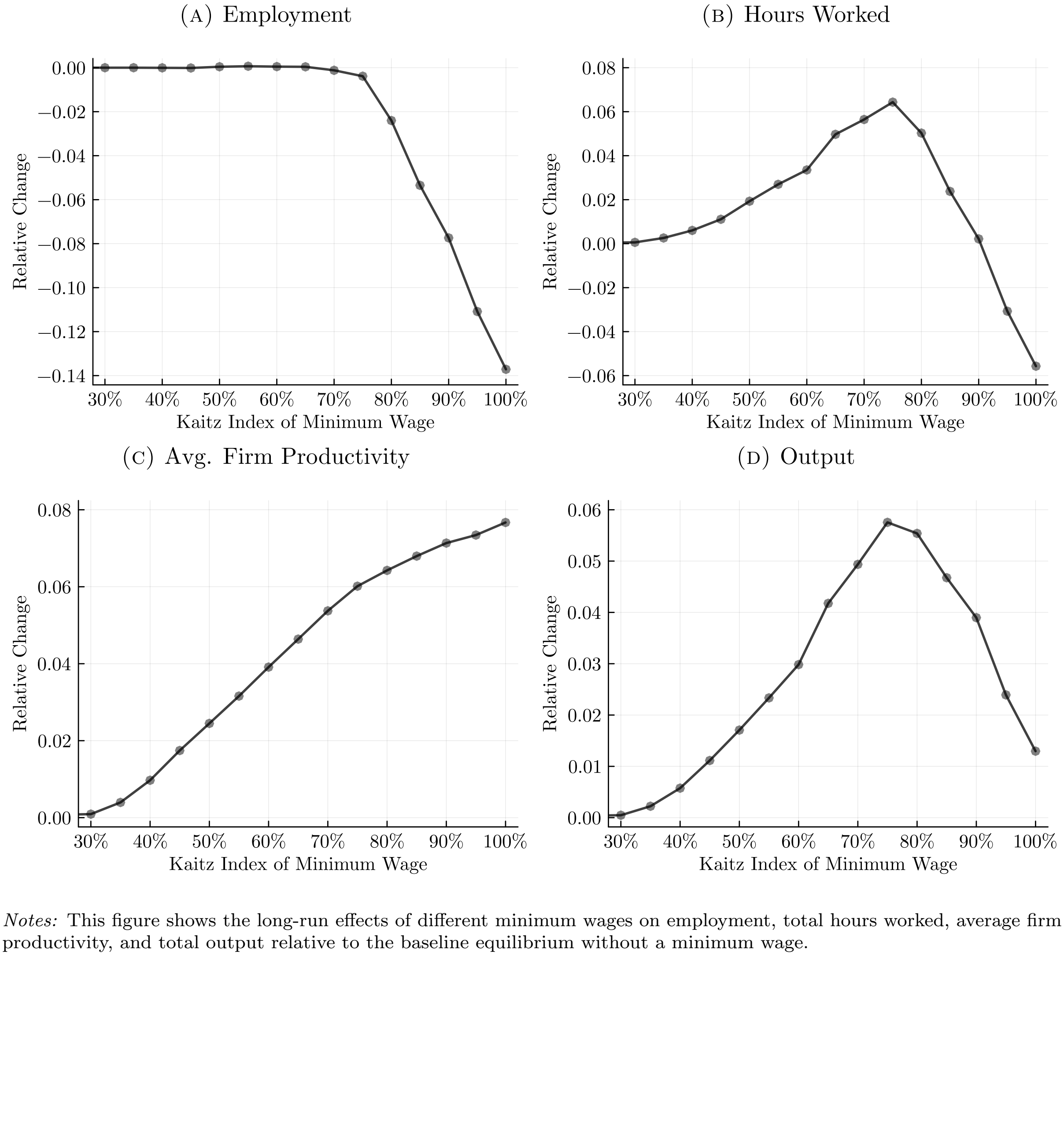
Panel C shows that average firm productivity also increases in the minimum wage. Conditional on worker skill, minimum wages hit low productivity firms harder than high productivity firms such that vacancy posting reacts asymmetrically.[45] At a Kaitz index of 65%, average firm productivity is up by 4.6%. Because of this increase in average firm productivity, the minimum wage does lead to a massive increase in the aggregate labor share which increases by only 1.6 percentage points despite an increase in average earnings of 6.8% (Appendix Figure A.8).

The increase in hours worked and average firm productivity implies that the composition of jobs improves in the sense that average output per job increases. As a result, Panel D shows that total output increases even though the number of jobs does not. At a Kaitz index of 65%, where the number of jobs is virtually the same as in the baseline, improved job composition

[44]The baseline full-time median wage is 18.9€ such that a Kaitz index of 60% corresponds to a minimum wage of 11.3€. All nominal minimum wages are in 2014 Euros. Appendix Figure A.6 shows the relationship between the Kaitz index and the bite of the minimum wage, i.e., the number of jobs paying less than the minimum wage before the reform.

[45]Appendix Figure A.7 shows the asymmetric effect on profit shares and vacancy posting across firms of different productivity for selected minimum wages.

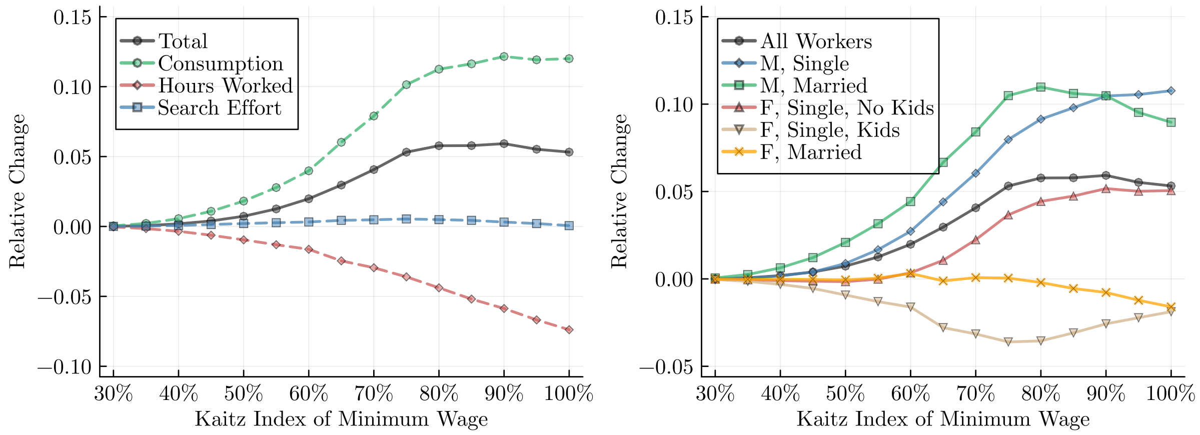
Figure 5: Long-Run Employment and Reallocation Effects



Notes: This figure shows the long-run effects of different minimum wages on employment, total hours worked, average firm productivity, and total output relative to the baseline equilibrium without a minimum wage.

Figure 6: Aggregate and Heterogeneous Welfare Effects

(a) All Workers (b) By Sociodemographic Groups



Notes: This figure shows how the minimum wage changes average worker welfare for different skill groups and demographic types in terms of consumption equivalents relative to the baseline equilibrium without a minimum wage. See Appendix B for details on the computation of (component-specific) consumption equivalent welfare effects.

through reallocation leads to a total output increase of 4.2%. As hours worked, output peaks at a Kaitz index of 75%. The monotone increase in average firm productivity implies, however, that output declines slower than hours worked as the minimum wage starts to destroy a substantial share of jobs.

5.2 Heterogeneous Welfare Effects

Figure 6 shows how the minimum wage changes worker welfare in terms of consumption equivalents. The black line in Panel A shows that the average welfare of all workers increases by 3.0% at a Kaitz index of 65% and continues to increase until a Kaitz index of 80%. Despite the fact that employment falls rapidly as the minimum wage increases further, the subsequent decline is slow. This is because large welfare losses of relatively few low skill workers are offset by minimum wage gains in the more densely populated middle of the skill distribution. In addition, government transfers put a lower bound to the welfare drop of low-skill workers who are no longer able to find a job.

The other lines in Panel A show the average consumption equivalent welfare change induced by each of the three components of lifetime utility—consumption, search and hours worked—while holding the other two components at their respective baseline levels.[46] While workers benefit from the change in wages and ultimately consumption utility, they lose from the increase in average hours worked. At a Kaitz index of 65%, the increase in earnings raises average welfare by 6.0%, whereas the increase in hours worked reduces average welfare by 2.5%. Minimum wage induced changes in search effort do not result in significant welfare changes.

[46]See Appendix B for details on the computation of (component-specific) consumption equivalent welfare effects.

Combined with differences in the disutility of hours worked, this translates into substantial heterogeneity in welfare effects across demographic groups (Panel B). While men and single women without kids benefit from higher minimum wages, married women's welfare is unaffected and single women with kids actually experience welfare losses. Appendix Figure A.10 shows that this is driven by differences in the hours-related component of welfare, which is roughly unchanged for men and declines substantially for women.[47]

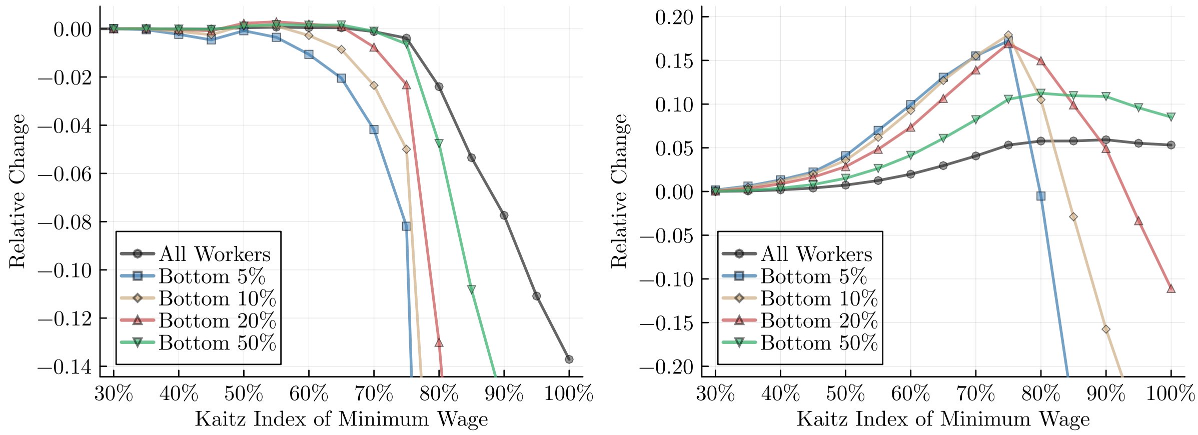
At this point, it is worthwhile to recall that workers in the model can only indirectly choose hours worked by choosing the type of job they accept and engaging in on-the-job search. This may imply that the model overestimates heterogeneity in welfare effects across groups that differ in their disutility of hours worked, because firms' vacancy posting reacts to the average acceptance decisions and on-the-job search behavior of all workers in the economy. To the extent that the average demand for low-hours jobs declines more strongly than that of single women with kids, who are by far the smallest group of workers and have limited impact averages, they may be more frequently mismatched to high-hours jobs which lowers their welfare. Nevertheless, as hours-related disutility of a given job is unaffected by the minimum wage, all demographic groups will have an incentive to work longer hours and contribute to the decline in the share of marginal vacancies. Hence, even with directed search more workers would direct their search effort away from marginal jobs after a minimum wage hike. To avoid discrete jumps in the effect of the minimum wage on hours worked. Single women with kids could then look for a less strong increase in hours worked which would reduce the consumption-related welfare gains and decrease the hours-related welfare losses. Such a setup would be computationally much more demanding and difficult to discipline with the German administrative data which do not contain information on hours worked.

The second dimension of exogenous worker heterogeneity is human capital. Panel A of Figure 7 shows that, unsurprisingly, employment rates drop more quickly for low-skill workers. In particular, the share of employed workers within the bottom 5% of the skill distribution falls significantly below its baseline value for Kaitz indices above 55%. At a Kaitz index of 65%, the employment rate in this group of workers is down by about 2%. Panel B shows how welfare effects differ across these skill groups. While low-skill workers benefit the most from increasing the minimum wage, they also suffer the most if the minimum wage is set so high that they become unemployable.

[47]Appendix Figure A.9 shows that demographic heterogeneity in employment rates and income is limited.

Figure 7: Employment and Welfare Effects by Worker Skill

(A) Employment (B) Welfare



Notes: This figure shows how the relative change in employment and worker welfare in consumption equivalents for different skill groups relative to the baseline equilibrium without a minimum wage. See Appendix B for details on the computation of consumption equivalent welfare effects.

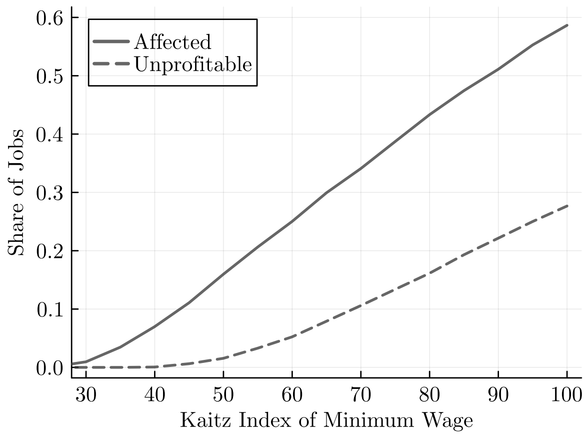
5.3 Transition Dynamics

I now analyze how the dynamics of minimum wage effects in frictional labor markets. The steady state comparisons show that reallocation is crucial to understand the effects of increasing minimum wages. While search frictions are the fundamental reason why the minimum wage can improve production efficiency, they also imply that any reallocation process takes time such that short- and long-run effects may differ substantially. This is particularly true for labor markets characterized by low job finding rates, as is the case in Germany and most other European countries (Hobijn and Şahin, 2009).[48] As a benchmark, I first study sudden transitions from the baseline equilibrium without a minimum wage to the new equilibrium with a minimum wage.

The starting point for non-trivial transition dynamics is the fact that some combinations of worker and firm productivity do not produce enough output to cover the wage costs implied by the minimum wage. These matches become unprofitable for firms and result in initial job destruction. Figure 8 shows how many jobs in the baseline equilibrium will be affected, i.e., pay a wage below the new minimum wage, and will become unprofitable for firms, i.e., have (marginal) productivity below the new minimum wage. The higher the minimum wage, the more jobs will be affected and destroyed following the minimum wage hike. While initial job destruction is not important for moderate minimum wages with Kaitz indices below 60.0%, the share of unprofitable jobs surpasses 5% for higher minimum wages. At a Kaitz index of 65%, where the long-run employment effect is essentially zero, 7.9% of all jobs are destroyed initially. These workers become unemployed and have to find a new job at a more productive employer.

[48] In Section 5.4, I will show how the transitional dynamics change when labor market turnover rates in the baseline economy are higher.

Figure 8: Bite of the Minimum Wage: Affected and Unprofitable Jobs



Notes: This figure shows the share of jobs in the equilibrium without a minimum wage that will be affected, i.e., pay a wage below the new minimum wage, and will become unprofitable for firms, i.e., have (marginal) productivity below the new minimum wage, for different Kaitz indices.

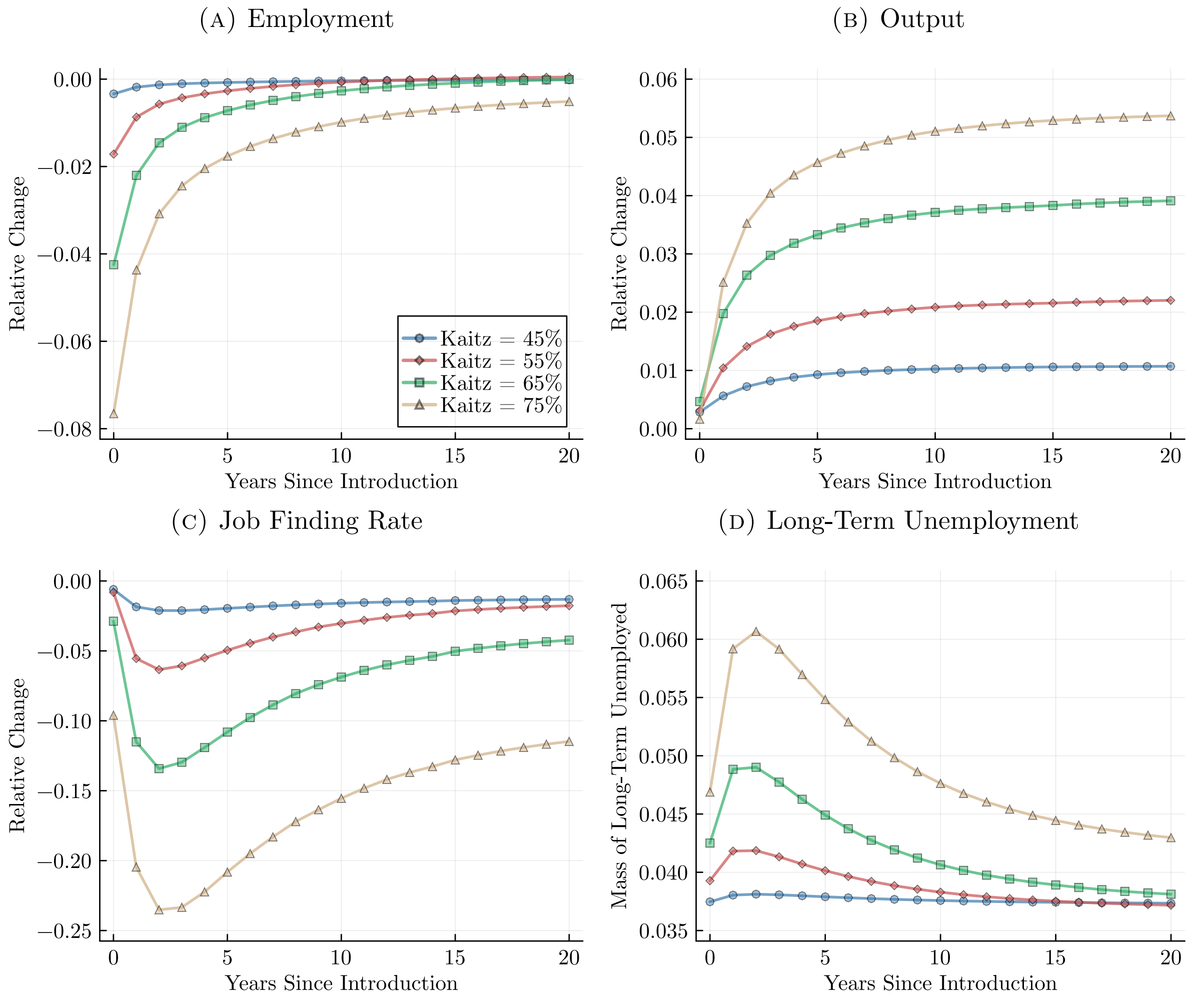
Figure 9 shows how employment and output evolve following the sudden implementation of different minimum wages. Panel A shows that employment falls significantly at the time the minimum wage is imposed. The initial decline in the number of jobs for Kaitz indices of 55%, 65% and 75% equals 2.0%, 4.2% and 7.7%, respectively. And even five years after the introduction, employment is still significantly below its new long-run value for Kaitz indices of 65% and 75%. While the time of convergence is closely related to the size of the initial drop, convergence is further prolonged because job finding rates temporarily decline below the new long-run value (Panel C) because a sizeable share of workers slide into long-term unemployment (Panel D) where search effort is less efficient and job finding rates are lower compared to short-term unemployment.[49]

A key insight of the analysis is that the difference between the short- and long-run effects is a non-linear function of the minimum wage hike. For low and moderate minimum wages with Kaitz indices below 50%, only very few jobs are destroyed initially and the transition path is relatively smooth. For higher minimum wages, however, the number of jobs that are destroyed on impact increases disproportionately in the minimum wage and the long-run gains of reallocation are preceeded by years of elevated unemployment.

Starting from a Moderate Minimum Wage. Thus far, I use the equilibrium without a minimum wage as the baseline and consider hypothetical reforms that implement the new minimum wage in one step. One natural question to ask is whether the stark difference between

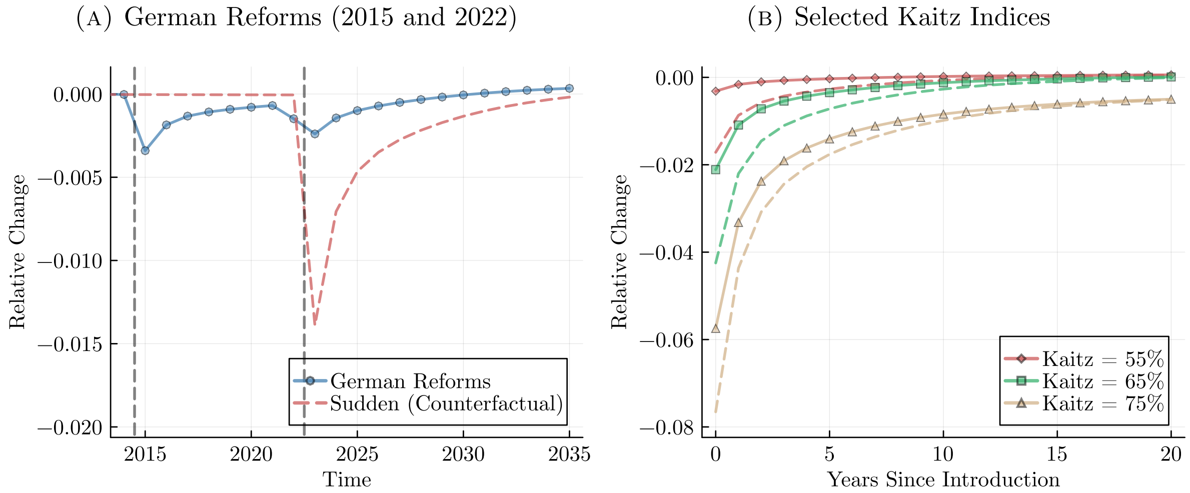
[49]Search efficiency during long-term unemployment is more the 60% less efficient ($\phi_{lu}/\phi_{su} = 0.38$) and the average job finding rate out of long-term unemployed is only about a quarter of the job finding rate out of short-term unemployed in the baseline equilibrium without a minimum wage. Appendix Figure A.11 shows the evolution of employment for a Kaitz index of 65% if the average job finding rate is counterfactually set to its long-run value immediately after the initial drop in employment. In this case, convergence takes only five years and the largest speed gain happens after three years.

Figure 9: Dynamic Minimum Wage Effects



Notes: This figure shows the dynamic effects of selected minimum wage reforms on employment, output, the average job finding rate (out of unemployment) and the mass of long-term unemployed workers relative to the baseline equilibrium without a minimum wage. In all scenarios, I assume that the minimum wage hike is announced two quarters before it becomes binding.

FIGURE 10: Dynamic Minimum Wage Effects: German Reforms and Moderate Initial Minimum Wage



Notes: Panel A of this figure shows the evolution of employment as implied by the German minimum wage introduction and the minimum wage reform in 2022. The red dashed line shows the counterfactual employment effects had the second reform been implemented without the first. Panel B shows the dynamic employment effects following minimum wage hikes of different magnitudes and for different baseline equilibria. The dashed lines correspond to the case where there is no minimum wage before the reform and the solid lines depict the case where the baseline is the stationary equilibrium with a Kaitz index of 45%.

short- and long-run effects of high minimum wages disappears when starting from a moderate minimum wage. After all, many countries—including Germany today—already have a binding minimum wage in place.

In Germany, the initial introduction of a minimum wage in 2015 was followed by a second big reform that raised the minimum wage to 10.45€ in the third quarter and 12€ in the fourth quarter of 2022, affecting around one fifth of all jobs.[50] Using average wage growth of full-time workers to deflate nominal minimum wages, this implies that the Kaitz index increased to 53.4%.[51] In Panel A of Figure 10, I study the employment effects induced by the sequence of minimum wage reforms in Germany and contrast the resulting employment dynamics to the counterfactual scenario where the minimum wage is increased from zero to the terminal without the first reform (red dashed line). According to the model, the second reform did not have a substantial negative effect on employment. In 2023, employment was only 0.24% lower compared to the baseline without a minimum wage. Due to a lack of data, the reform is yet to be evaluated empirically. However, the model's predictions are in line with the fact that aggregate data provide no indication that the second reform had a substantial negative effect on employment. In contrast, the employment effect would have been much larger—1.39% or about half a million jobs—if the second reform in 2022 had been implemented without the first reform 2015.

[50]Following the introduction of the minimum wage in 2015, the minimum was actually raised multiple times (roughly every two years) in a way that kept the real minimum wage constant. For simplicity, I abstract from these marginal intermediate increases and focus on the increases in 2022.

[51]According to aggregate data of the federal statistical office, average full-time wages grew by 19% between 2014 and 2021 (from 20.68€ to 24.60€). Source: Statistisches Bundesamt (2014, 2021).

Panel B of Figure 10 shows the employment effects for transitions toward equilibria with higher minimum wages when the Kaitz index in the initial equilibrium is 45% rather than 0%.[52] While starting points differ, I always show the relative change with the zero minimum wage equilibrium as the baseline for both cases. For a terminal Kaitz index of 55%, the initial spike in unemployment is almost gone. However, for Kaitz indeces of 65% and 75%, the initial spike remains substantial. The reason behind this is that long-run reallocation effects are non-linear (see Figure 8) in the minimum wage such that equilibria with moderate minimum wages between 30% and 50% are closer to the baseline without a minimum wage than to the terminal equilibria with minimum wages above 60%.

Gradual Implementation. Based on these results, I now ask whether the stark difference between short- and long-run effects of high minimum wages disappears when the minimum wage is implemented gradually over several years. To study this question, I focus on the transition toward the stationary equilibrium with a Kaitz index of 65%, which has virtually the same employment rate as the baseline equilibrium without a minimum wage. In particular, I assume that the minimum wage first jumps from zero to a Kaitz index of 45% and then increases linearly over $T \in \{8, 20, 40\}$ quarters until it reaches the terminal Kaitz index of 65%.[53]

Panel A of Figure 11 shows that the dynamic employment response changes drastically when the minimum wage is phased-in gradually. While employment drops by over 4% following a one-time minimum wage hike, the initial drop is more than halved with a phase-in period. With a phase-in of five years, employment drops by only 0.5% during the phase-in period and then increases again. A phase-in of 10 years leads to the same initial drop in employment, but slightly lower disemployment effects thereafter. Appendix Figure A.13 shows that the absence of transitional disemployment effects is due to the fact that the increase in long-term unemployment and therefore the decline in job finding rates does not overshoot during the transition.

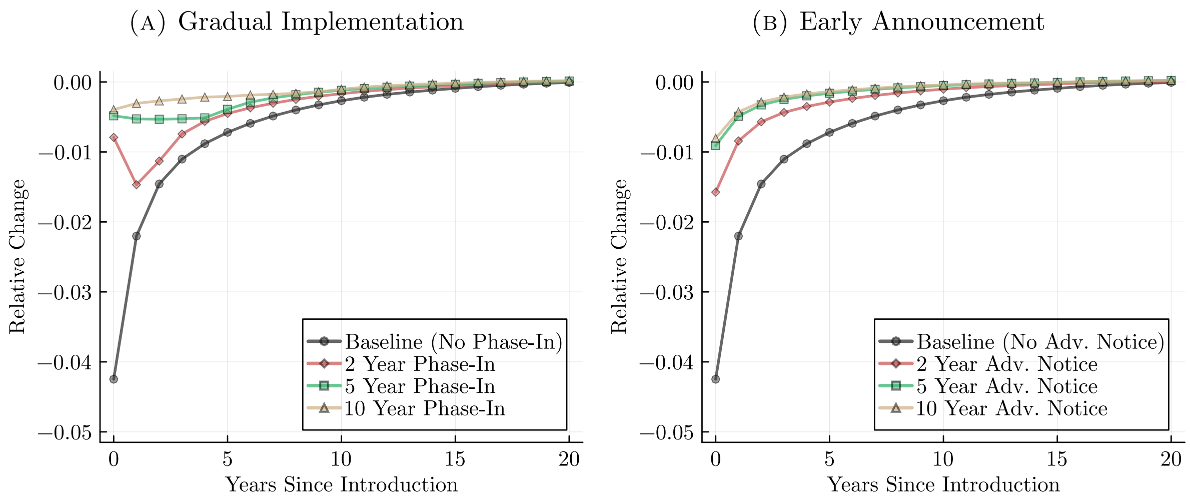
Appendix Figure A.14 shows the dynamic employment effects when the gradual minimum wage increase starts from the equilibrium with a Kaitz index of 45%. This slight change in the starting point removes the small initial employment drops that are still visible in Panel A of Figure 11. For a five year phase-in period, employment declines (by 0.2%) during the first five years and then starts to increase again—precisely as in the empirical analysis of recent large and gradual minimum wage increases by Clemens and Strain (2021).

But why does a gradual implementation of the minimum wage reduce transitional unemployment? First, forward-looking firms anticipate future increases in the wage bill due to higher minimum wages. This affects firms' vacancy posting before the minimum wage reform through the expected value of filling a vacancy. As low-productivity firms will be hit harder by the minimum wage, they reduce their vacancy posting more than high-productivity firms, triggering reallocation away from jobs that will become less profitable in the near future. Hence, once

[52]This value is not only the initial Kaitz index in Germany after the introduction in 2015, but also right in the middle of the range of current minimum wage levels in developed countries (Dube, 2019) or US state-level minimum wages (Cengiz et al., 2019).

[53]Appendix Figure A.12 shows the time series of the Kaitz index for these gradual minimum wage reforms.

FIGURE 11: Dynamic Employment Effects: Gradual Implementation and Early Announcement (Kaitz Index = 65%)



Notes: Panel A of this figure shows the dynamic employment effects following minimum wage reforms that first increase the minimum wage to 45% and then gradually increase it further to a Kaitz index of 65%. Panel B shows the effects for one-time increases in that are announced in advance.

the minimum wage becomes binding, fewer jobs will become unprofitable. Panel B shows that simply announcing a one-time reform $T \in \{8, 20, 40\}$ quarters in advance substantially softens the initial employment drop. Second, gradual implementations spread out the destruction of unprofitable jobs over several years. This reduces total job destruction, since workers who are laid off after the first increase can find another job at a firm that will not be destroyed by the next increase. Appendix Figure A.15 shows that disemployment effects are halved when the five year phase-in is a sequence of unanticipated reforms.

5.4 Robustness Analysis

Internally Estimated Parameters. In order to assess the robustness of the results with respect to changes in the model's internally estimated parameters, I vary each parameter individually around its estimated value and re-compute the long-run employment and output effects of different minimum wages.[54] Figure A.16 shows that the main results are robust. In particular, there are no substantial disemployment effects for minimum wages with Kaitz indices up to 65% and there is little variation in the disemployment effects for higher minimum wages.

Note that this also holds for the convexity of the vacancy posting cost function, κ_2, which prevents high-productivity firms from growing without bound and thus disciplines the variation in firm size in the model. Recall that κ_2 also determines the elasticity of vacancy posting with respect to the expected value of opening a vacancy, $1/(\kappa_2 - 1)$. Higher values of κ_2 lead to weaker responses of vacancy posting to changes in the expected value of posting a vacancy. However, Appendix Figure A.17 shows that changes in κ_2 do not significantly impact employment effects. If anything, higher values of κ_2 actually lead to slightly stronger disemployment effects. As in

[54] I vary each parameter around its estimated value as described in Appendix D.

Engbom and Moser (2022), this is related to the asymmetry in the response of vacancy posting across heterogeneous firms (see Appendix Figure A.7). A lower elasticity leads to less employment cuts by low-productivity firms but also reduces reallocation toward high-productivity firms. Quantitatively, these effects largely offset each other.

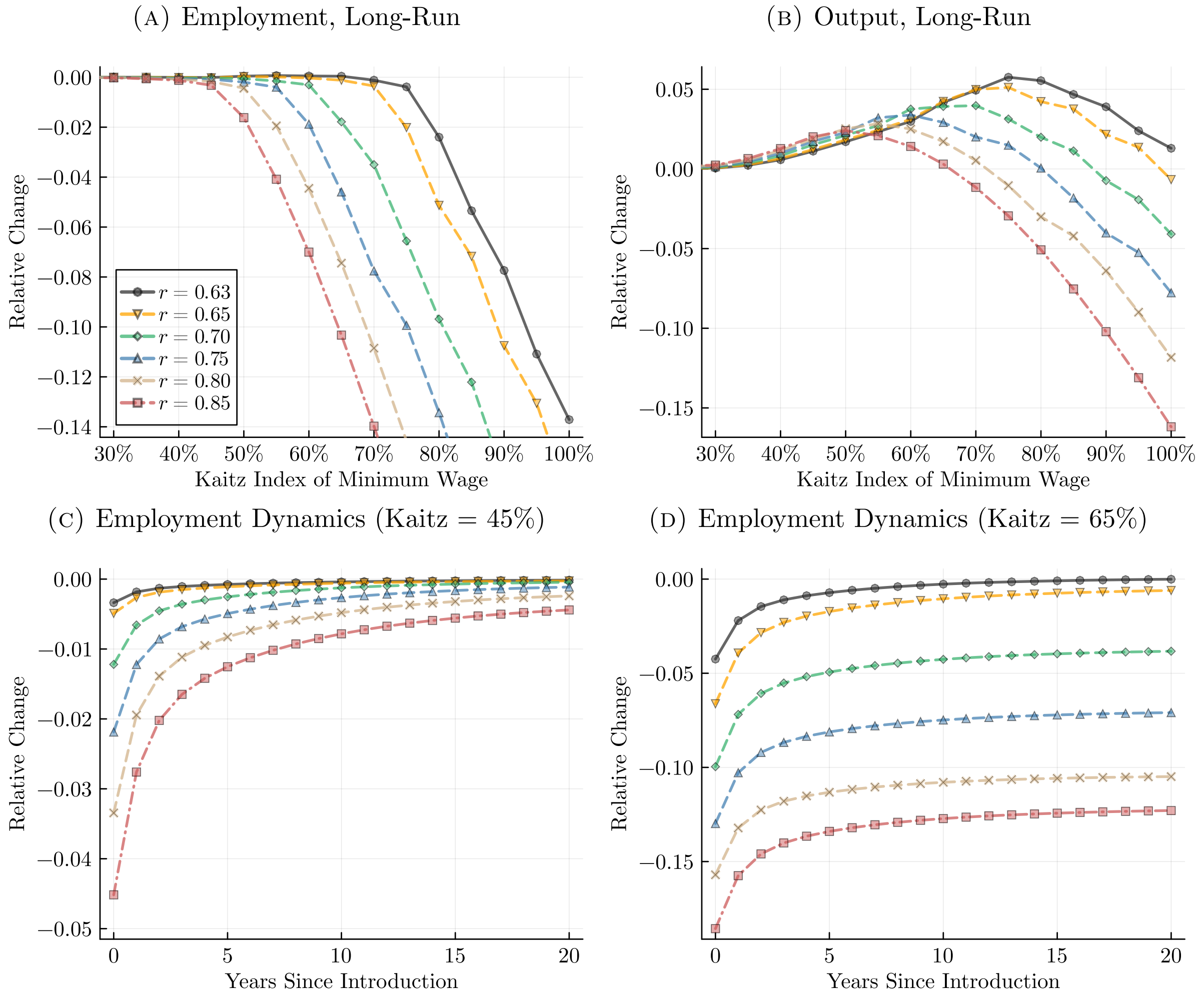
In contrast to the internally estimated parameters, the employment and output effects of minimum wages are more sensitive to changes in two important parameters: the elasticity of matches with respect to vacancies, ξ, and the wage piece rate, r.

Wage Markdown. Panels-A and B of Figure 12 show the long-run employment and output effects of different minimum wage hikes for higher piece rates. Higher piece rates reduce the gap between marginal productivity and wages and hence firms' cushion against minimum wage hikes. Interestingly, higher piece rates do not change the overall shape of the relationship between the minimum wage and employment. They simply shift the curve to the left such that disemployment effects kick in for lower minimum wages. Before these turning points, employment effects are negligible for all piece rates, and after these turning points, employment deflines at a similar rate in the minimum wage. For example, with a baseline piece rate of $r = 0.75$, the decline in employment starts between Kaitz indices of 50% and 55%. With a piece rate of $r = 0.85$, minimum wages with Kaitz indices above 45% lead to significant long-run employment drops. As disemployment effects start earlier, the output-maximizing minimum wage shifts downwards as the piece rate increases (Panel B). In addition, the maximum long-run output gain from reallocation is lower for higher piece rates since there is less room for reallocation before employment starts to drop.

Panel C shows how higher piece rates alter the dynamic employment effects of for a moderate Kaitz index of 45% for which we have quasi-experimental evidence from the German minimum wage introduction. While the long-run employment effects at a Kaitz index of 45% are small for all piece rates below 0.85, the short-run effects differ substantially. In particular, higher piece rates increase the intial drop in employment. The intuition behind this result is rather straight-forward: If a given minimum wage does not destroy jobs, these jobs must still be profitable for firms, which implies that the wage increase was smaller than the wage markdown relative to productivity. In other words, a given ratio of minimum to median wage (Kaitz index) that requires similar wage increases leads to larger employment effects when the piece rate is higher. According to the model, piece rates of $r \geq 0.7$ lead to a drop in employment of over 1% in the year of the reform, implying an own-wage employment elasticity of over 0.5 in the first year and elasticities of over 0.2 in the first five years after the reform. Through the lens of the model, the empirical evidence on the employment effects of the German minimum wage introduction in 2015 suggests us that piece rates above 0.7 are at odds with the data as they cannot jointly reproduce an empirically plausible wage distribution (approximated by the Kaitz index) and the small employment effects of the reform.

Note, however, that in the model, firms are allowed to fire workers in case the minimum wage renders the job unprofitable. To the extent that this does not hold in the data, the

Figure 12: Minimum Wage Effects for Higher Piece Rates



Notes: This figure shows the long-run employment and output effects, and the dynamic employment effects of different minimum wage hikes for different assumptions about the piece rate in the absence of a minimum wage. In particular, starting from the baseline version with $r = 0.63$, I increase r to $r' \in \{0.65, 0.7, 0.75, 0.8, 0.85\}$, leaving all other model parameters constant. I then solve the equilibrium for different Kaitz indices of the minimum wage and compare it to the respective baseline without a minimum wage. Panels A and B show the long-run employment and output effects and Panel C and Panel D show the dynamic employment effects for a minimum wage reform that increases the Kaitz index from 0% to 45% and 65%.

empirically observed null effect on employment after the German minimum wage introduction in 2015 could be consistent with somewhat higher piece rates. In that case, the quantitative results in this paper are an upper bound on the employment and reallocation effects of minimum wages. Importantly, the qualitative insight that reallocation toward more productive firms is an important force that mitigates the long-run disemployment effects of minimum wages is unaffected by the choices of piece rate.

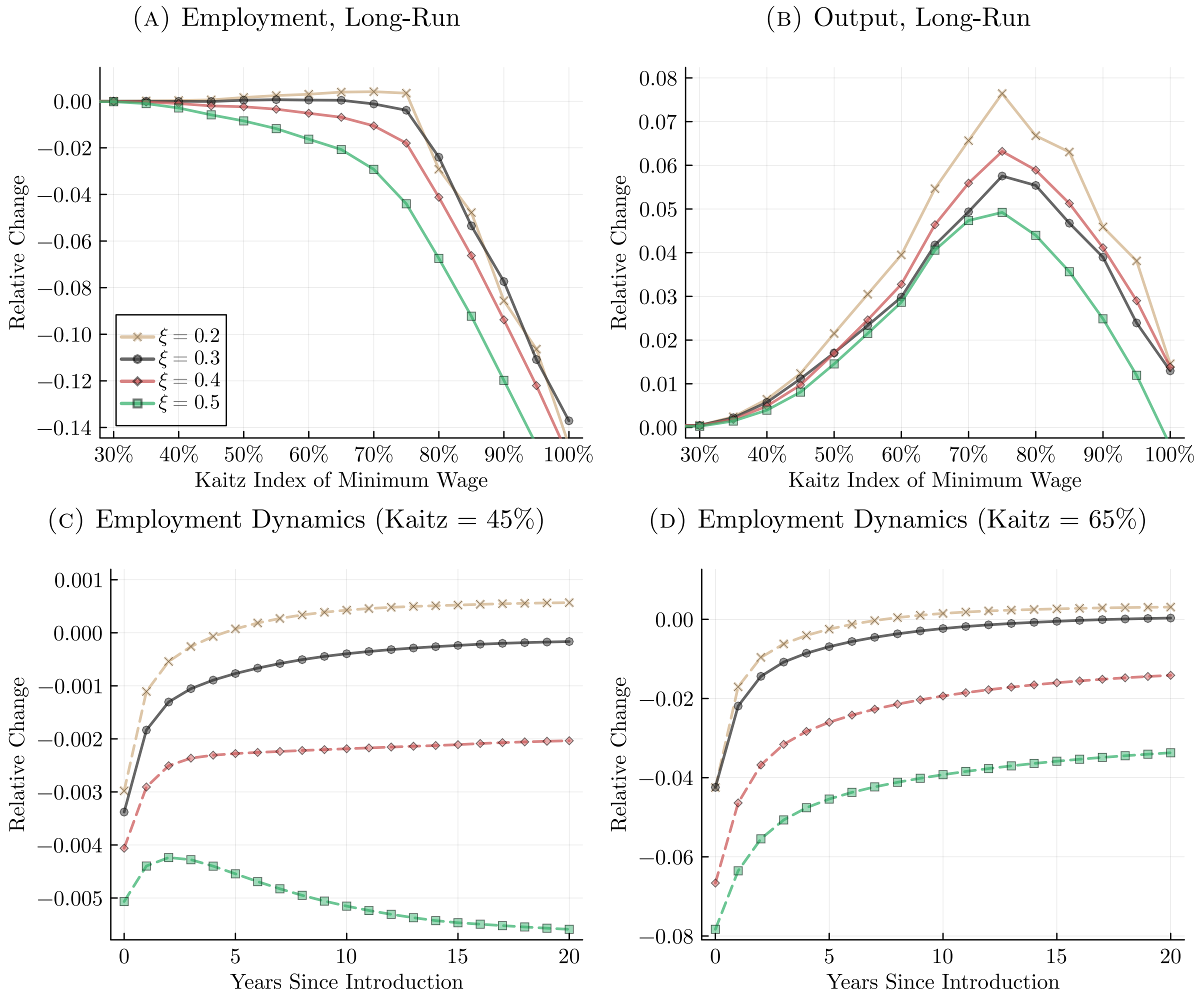
Matching Elasticity. In the search-and-matching framework, the elasticity of matches with respect to vacancies, ξ, determines how changes in firms' vacancy posting affect the number of matches and thus the number of jobs in the economy. Panel A of Figure 13 shows that the disemployment effects of minimum wages are more pronounced for higher values of ξ. In particular, the more responsive the number of matches is to changes in vacancies, the smoother the decline in employment as the minimum wage increases. For an elasticity of 0.5, the decline in employment starts at a Kaitz index of 40% and exhibits less of a sharp kink. Reducing the elasticity to 0.2 leads to small positive employment effects up to a Kaitz index of 75% followed by a sharp decline thereafter.

In contrast to the employment effects, output effects are non-monotone (Panel B) in the vacancy-elasticity of matches. While output gains are slightly larger for $\xi = 0.4$ compared to the baseline of 0.3, they are smaller for $\xi = 0.5$. This is because the positive effects on average productivity are stronger for higher values of ξ which partially offsets the stronger decline in (low-skill) employment.

Panel C shows the dynamic employment effects for a Kaitz index of 45% for different values of ξ. Interestingly, for $\xi = 0.5$, the long-run employment effects are large enough such that they are more negative than the short-run effects. After the initial drop in employment because of minimum-wage induced layoffs, two opposing forces are at play. First, reallocation toward more productive firms pushes employment upwards as some of the initially released workers find a job at another firm. Second, the long-run decline in vacancy posting and worker-firm matches reduces the job finding probabilities of workers who lose their job after the minimum wage hike. However, without reallocation, the decline in employment would be more severe and monotone. In addition, the larger the minimum wage hike, the more important is initial job destruction. Panel D shows that long-run disemployment effects are less severe for all considered values of ξ when the minimum wage is increased from zero to a Kaitz index of 65% rather than 45%.

Labor Market Turnover. The final step of the analysis is to study how the dynamic effects of minimum wages change when search frictions are reduced. In particular, how important is transitional unemployment after large minimum wage hikes when baseline labor market turnover, i.e., job finding and destruction rates, is higher. Hobijn and Şahin (2009) estimate that, until 2004, average job finding rates in the US are eight times higher than in Germany and up to ten times higher compared to other countries in Western continental Europe. Other Anglosaxon and Nordic countries also have higher job finding rates (factor of 2 to 4 compared to Germany).

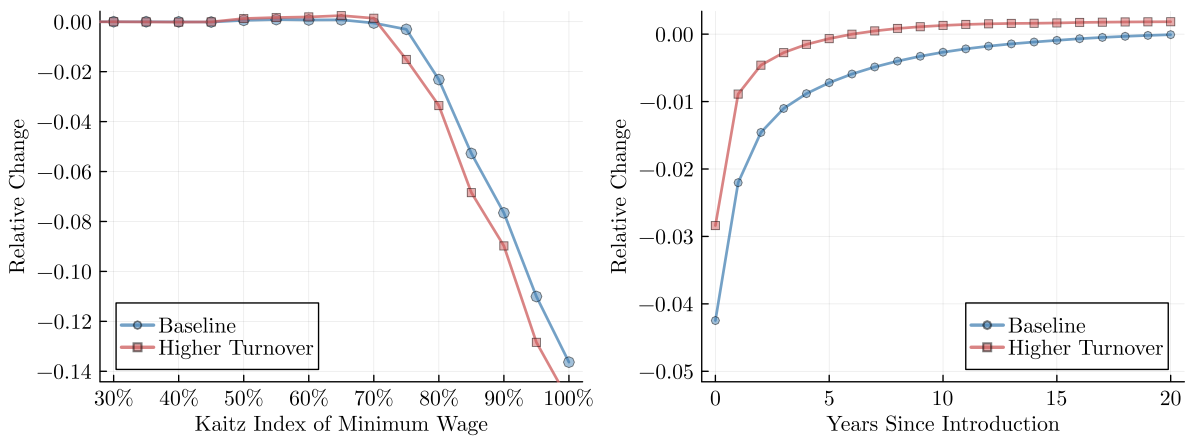
Figure 13: Minimum Wage Effects for Higher Vacancy-Elasticity of Matches



Notes: This figure shows the long-run employment and output effects for different Kaitz indices (Panels A and B), and the dynamic employment effects for a Kaitz index of 45% and 65% (Panel C and D) for different values of the elasticity of matches with respect to vacancies, $\xi \in \{0.2, 0.4, 0.5\}$ (baseline value of 0.3). All other model parameters are unchanged at their estimated values. I then solve the equilibrium for different Kaitz indices of the minimum wage and compare it to the respective baseline without a minimum wage.

FIGURE 14: Employment Effects with Higher Labor Market Turnover

(A) Long-Run Effects on Employment (B) Employment Effects (Kaitz Index 65%)



Notes: This figure shows the long-run employment effects for different minimum wage hikes and the dynamic employment effects following a minimum wage hike that increases the Kaitz index from 0% to 65% for the baseline model and a version with higher labor market turnover.

To assess the importance of search frictions and labor market turnover for the dynamics of minimum wage effects, I double workers' search efficiency, ϕ, and the job destruction rates, $\pi_{su|e_x}$. As intended, the average quarterly job finding rate roughly doubles from 16.8% to 32.7%, the share of long-term unemployed among all unemployed workers falls from 54.1% to 39.4%, whereas the overall employment rate remains roughly constant (92.7% to 93.2%).

Panel A of Figure 14 shows the long-run employment effects for different Kaitz indices in the baseline model and the re-parameterized version with higher labor market turnover. While the turning point for employment shifts slightly to the left, the long-run effects remain very similar. In contrast, Panel B shows the dynamic employment effects for a Kaitz index of 65%. With higher labor market turnover, the initial drop in employment is less severe and the recovery is faster. Nevertheless, transitional unemployment due to search frictions still plays a role in the first years after the minimum wage hike.

6 Conclusion

This paper presents a rich search-and-matching model with substantial worker and firm heterogeneity that is not only able to match important aspects of pre-reform micro data but can also replicate the available reduced form evidence on the German minimum wage introduction in 2015.

The analysis of counterfactually high minimum wages reveals an important role for two-dimensional reallocation effects following minimum wage hikes. In the long-run, reallocation toward more productive firms and full-time jobs can improve the composition of jobs while keeping employment roughly constant for minimum wages of up to 70% of the full-time median wage. In the presence of search frictions, however, reallocation takes time and the details of minimum wage reforms, i.e., the size of the minimum wage hike relative to the initial minimum wage level and the phase-in period, crucially shape the transition path. In order to avoid transitional unemployment, large minimum wage reforms need to be phased in over several years—a strategy that policy makers already follow in practice. The findings in this paper are particularly relevant for countries or states that currently have no or only a moderate minimum wage and are considering large minimum wage hikes.

A number of avenues for future research seem particularly fruitful given the potential benefits of high minimum wages. First, while wages in the tradable sector are relatively high in the absence of a minimum wage, high minimum wages raise the question at what point firms in the tradable sector decide to relocate to other countries. Just like for taxation, international cooperation may become important for minimum wage laws and should be studied more systematically. For example, the European Union recently passed a directive on adequate minimum wages that proposes a minimum wage with a Kaitz index of 60% across all member states in order to increase low-skill wages while also maintaining a level playing field for firms.

Second, more empirical research is needed in order to understand how higher minimum wages impact firms' and workers' investment decisions in order to endogenize the distributions of worker and firm productivity. In particular, to what extent will high minimum wages lead firms to replace labor with capital (Acemoglu and Loebbing, 2022; Hurst et al., 2022), and will workers find jobs at other, more productive firms or be re-assigned to other tasks? Similarly, how will high minimum wages affect human capital accumulation? On the one hand, higher minimum wages may decrease workers' incentives to invest in their education as wage differentials are reduced. On the other hand, the disappearance of jobs in low-skill segments of the labor market may increase workers' human capital accumulation.

Third, in order to better understand reallocation effects of minimum wages, it seems fruitful to investigate to what extent decreasing returns to scale in firms' production functions, but also firm entry and exit, affect the potential for reallocation and whether this will impact aggregate employment effects. However, adding decreasing returns to scale to a random search model requires strong assumptions to preserve tractability (Bilal et al., 2022).

Finally, the effects of the minimum wage interact with other labor market policies such as the design of unemployment insurance or earned income tax credits. As both unemployment benefits and minimum wages affect workers' surplus of employment, the optimal generosity of the social safety net and the level of the minimum wage should be determined jointly. The findings in this paper show that this point is particularly important in frictional labor markets where progressive tax-and-transfer schemes subsidize low-productivity jobs.

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A Additional Figures and Tables

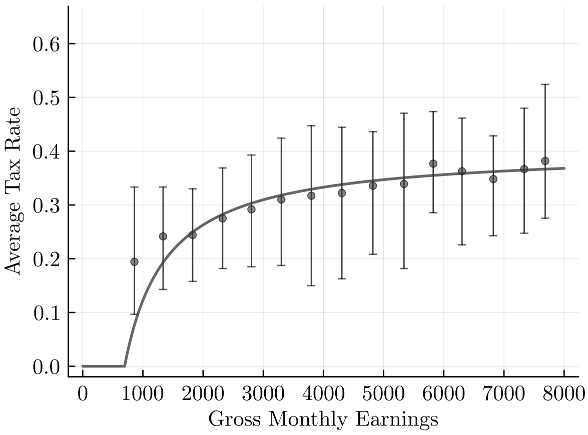
Table A.1: Distribution of Demographic Types

	$P_j = \Pr(j)$	$\Pr\big(g(j)\big)$	$\Pr\big(j \vert g(j)\big)$
Sociodemographics			
Men, Single	0.214	0.514	0.416
Men, Married	0.300	0.514	0.584
Women, Single, No Kids	0.168	0.486	0.346
Women, Single, Kids	0.046	0.486	0.095
Women, Married	0.272	0.486	0.560

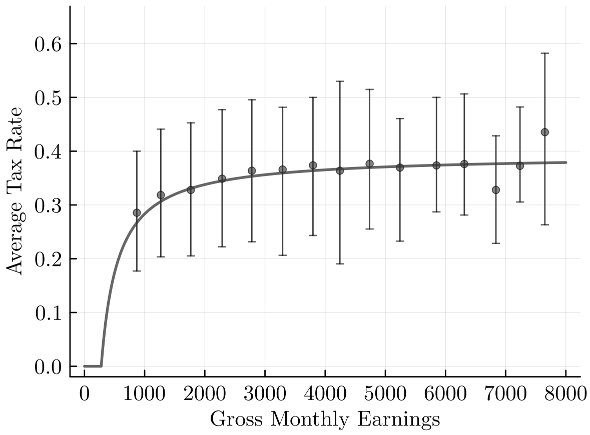
Note: The share of each sociodemographic group conditional on gender $g(j)$ is computed from the SOEP and then multiplied by the respective gender share in the SIAB data. Source: SOEP, SIAB, own calculations.

Figure A.1: Fit of Estimated Tax Functions

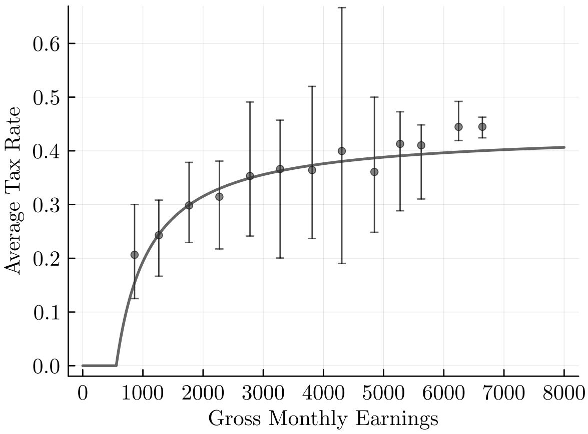
(a) Married Men



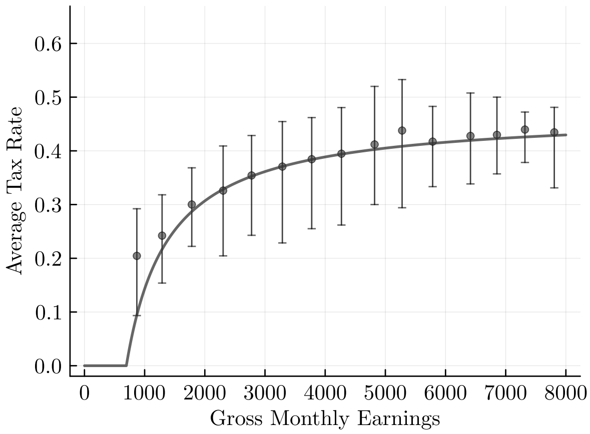
(b) Married Women



(c) Single Women with Kids

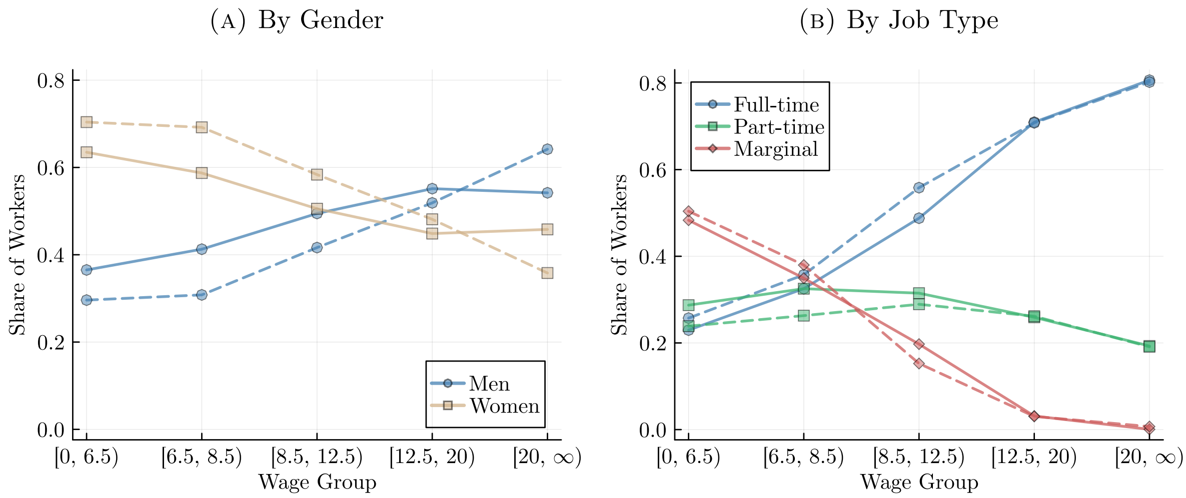


(d) Other Singles



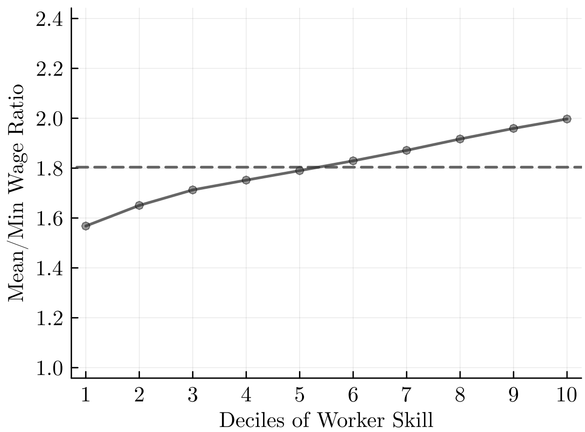
Notes: This figure shows estimated average tax functions as well as the mean average tax rate in various gross earnings bins. The spikes show the range between the 10^{th} and 90^{th} percentile of average tax rates in those bins. The average tax function is $T(y) = (1 - \tau^j)\max\{0, y - D^j\}/y$.

Figure A.2: Model Fit: Job Types and Gender across the Wage Distribution



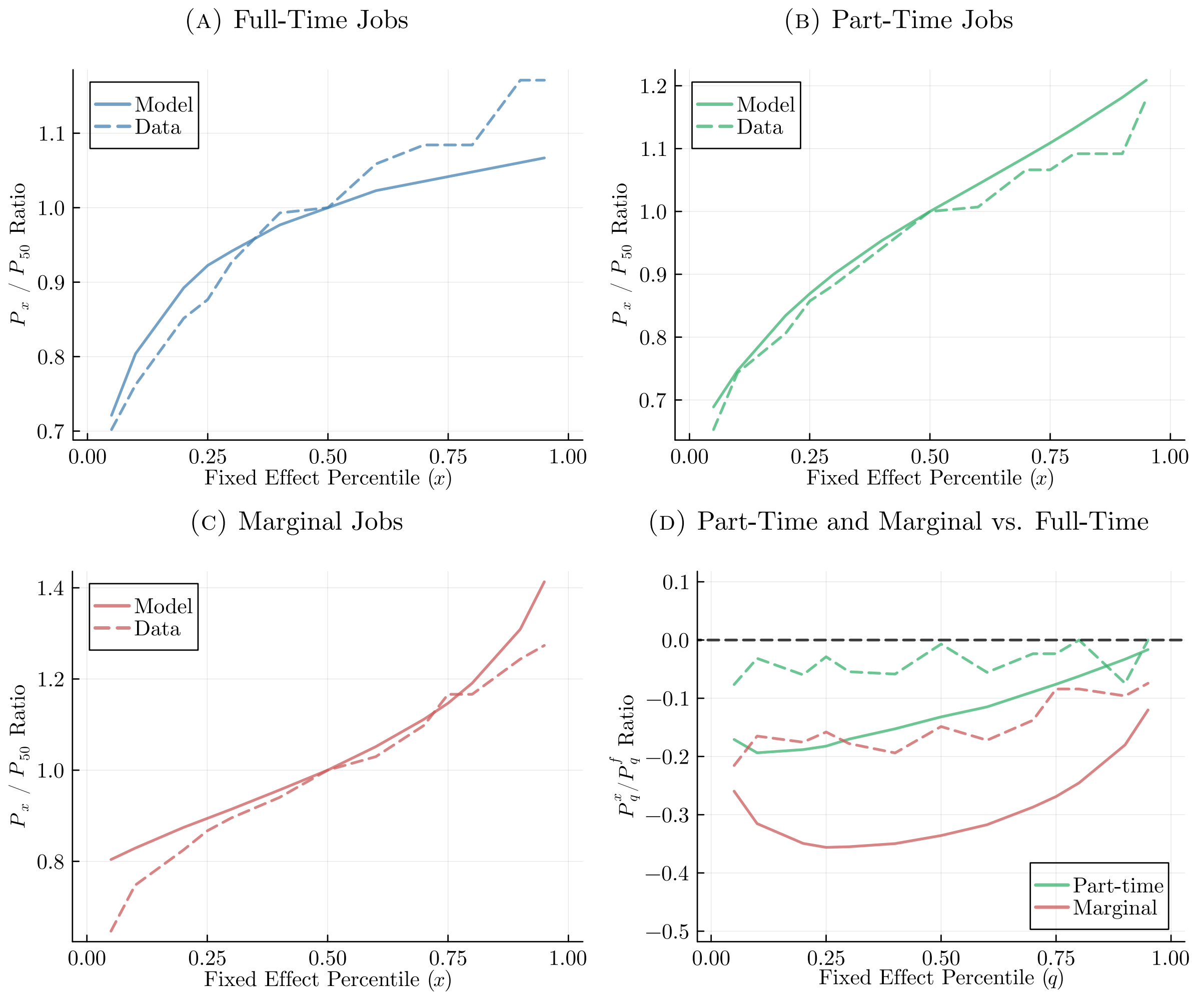
Notes: This figure shows the distribution of gender and job types (full-time, part-time, marginal) across different wage groups in the model (solid lines) and the data (dashed lines). The left panel shows the share of male and female workers in each wage group. The left panel shows share of full-time, part-time and marginal jobs in each wage group.

Figure A.3: Mean-to-Minimum Wage Ratios by Wage Deciles



Notes: This figure shows the average mean-to-minimum wage ratio as proposed by Hornstein et al. (2011) for different worker skill deciles in the model. To capture residual wage dispersion, I compute the mean-to-minimum wage ratio separately for each level of worker skill and then take the average within each skill decile. The dashed line shows the average over all skill levels.

Figure A.4: Model Fit: Firm Fixed Effect Distributions by Job Type

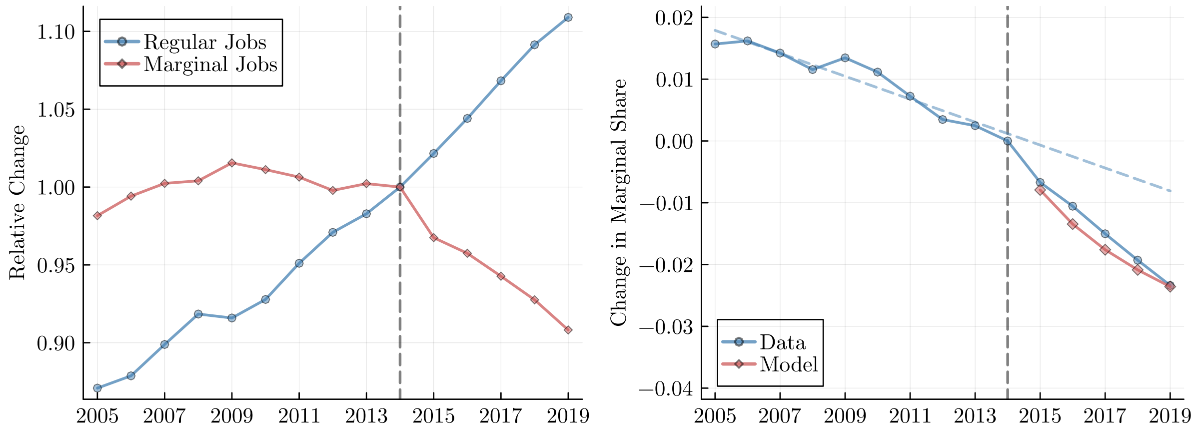


Notes: This figure shows the distribution of firm fixed effects among full-time (Panel A), part-time (Panel B), and marginal jobs (Panel C) in the model (solid lines) and the data (dashed lines). Panel D shows the ratio of selected percentiles of the firm fixed effect distribution for part-time and marginal jobs relative to the corresponding percentiles for full-time jobs.

Figure A.5: Marginal Jobs in the Data and the Model

(A) Number of Jobs by Type

(B) Share of Marginal Jobs

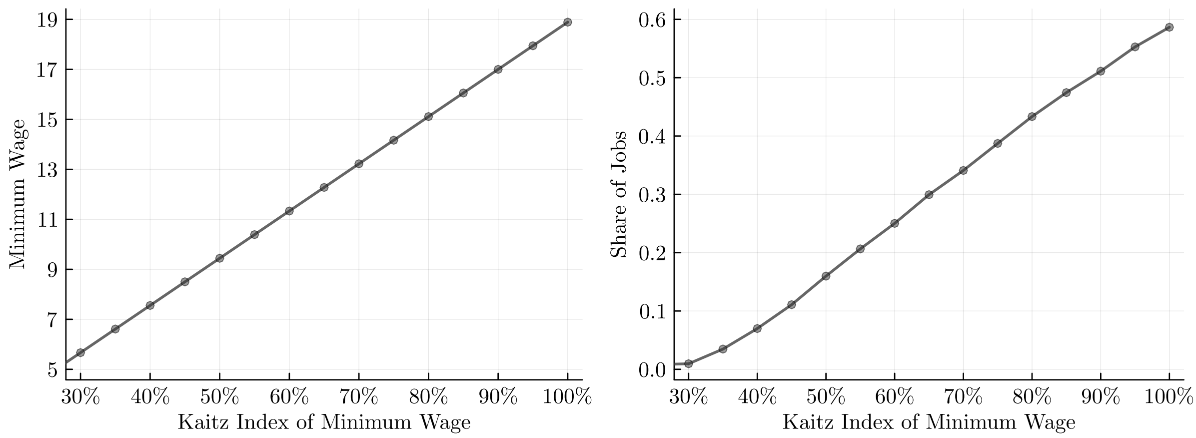


Notes: Panel A shows the change in the number of workers with regular (full-time and part-time) jobs and marginal jobs in Germany. Both series are normalized to one in 2014. Panel B shows the absolute change in the share of marginal jobs among all jobs, a linear trend fitted on the data before the minimum wage introduction, and the change in the share of marginal jobs predicted by the model. Both in the data and the model, averages across quarters are plotted. As in the IAB data used for estimation, I exclude secondary marginal jobs. Source: Federal Employment Agency.

Figure A.6: Minimum Wage Levels and Affected Jobs by Kaitz Indices

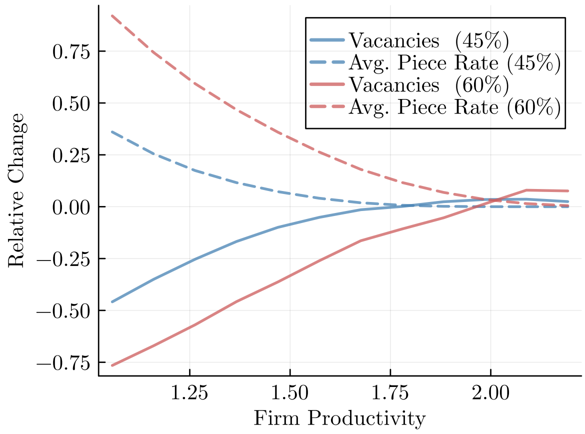
(A) Minimum Wage

(B) Affected Jobs



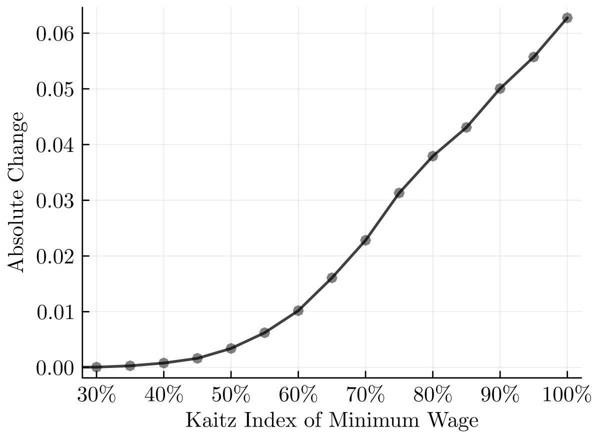
Notes: This figure shows the nominal minimum wage (2014 Euros) and the share of affected jobs, i.e., jobs paying less than the minimum wage before the reform, for different Kaitz indices.

Figure A.7: Asymmetric Minimum Wage Effects by Firm Productivity



Notes: This figure shows the change in average piece rates and vacancy posting across the distribution of firm productivity for Kaitz indices of 45% and 65% relative to the baseline equilibrium without a minimum wage. The average piece-rate is calculated using employment weights from the baseline equilibrium.

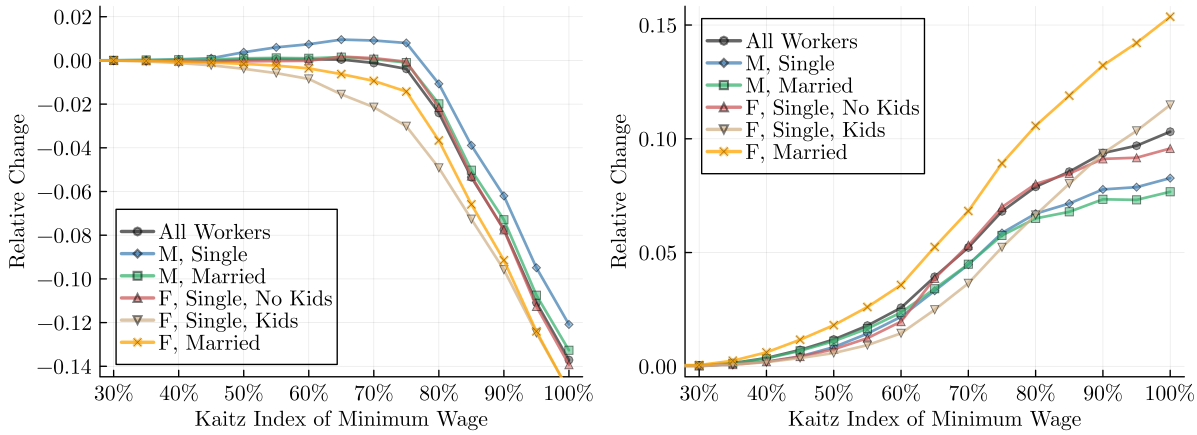
Figure A.8: Minimum Wage Effects on the Aggregate Labor Share



Notes: This figure shows the change in aggregate labor share relative to the baseline equilibrium without a minimum wage for different minimum wages.

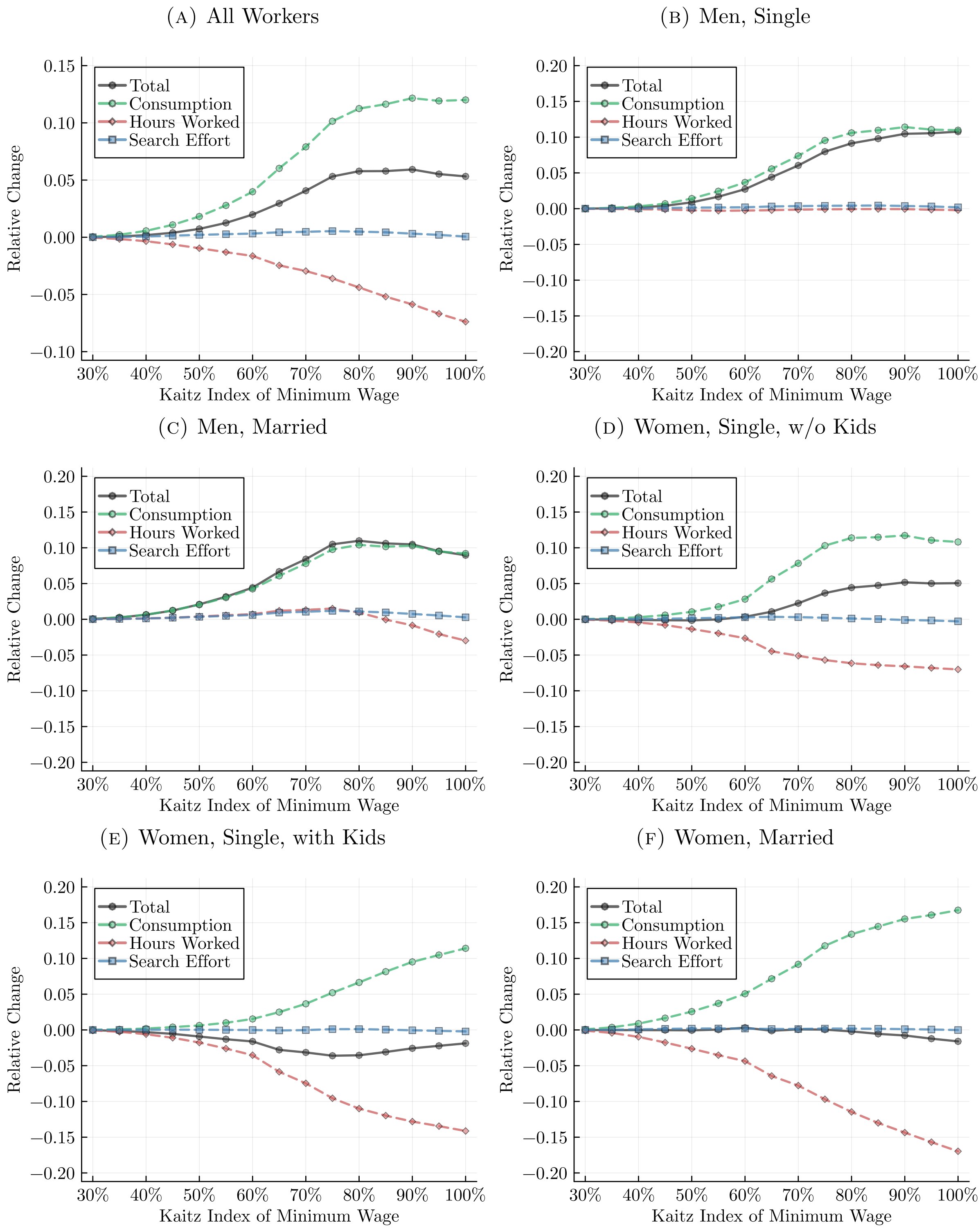
Figure A.9: Income and Employment Effects by Demographics

(a) Employment (b) Income



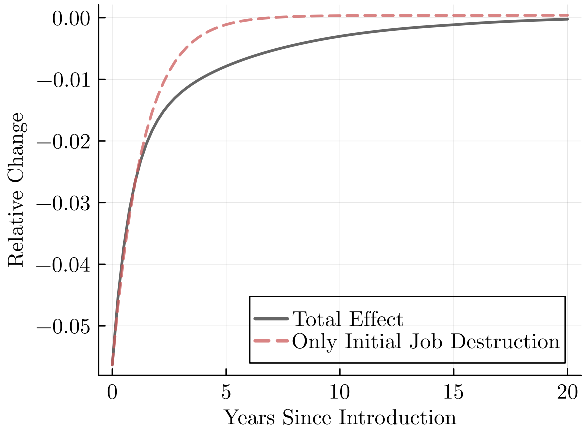
Notes: This figure shows the long-run relative change in average incomes and employment for different Kaitz indices relative to the baseline without a minimum wage across demographic groups.

Figure A.10: Decomposition of Consumption Equivalent Welfare Effects by Demographics



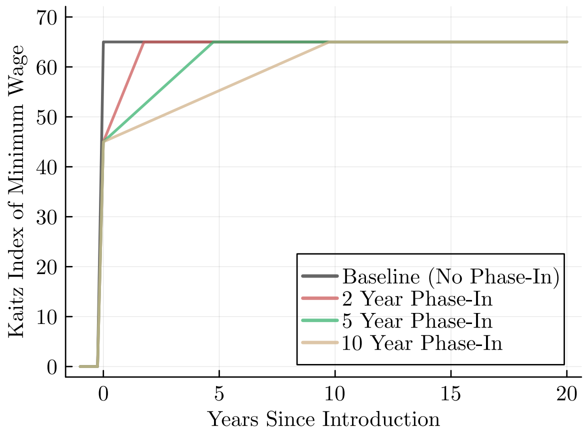
Notes: This figure shows the consumption equivalent change in total welfare and its components for different groups of workers in the long-run equilibrium with a minimum wage relative to the model without a minimum wage. Total lifetime utility equals $V = \sum_s \Pr(s_0 = s) \sum_{t=0}^{\infty} \sum_{s'} u(c(s_t), \ell(s_t), e(s_t)) \Pr(s_t = s' | s_0 = s) = V_c + V_e + V_\ell$ because u is additively separable in c, e and ℓ (equation 1). The consumption equivalent welfare change for a given component (consumption, hours worked, or search effort) is the relative change in consumption required to obtain the same change in lifetime utility if only that component of lifetime utility is taken from the new equilibrium. Due to non-linearities, the component-specific welfare changes do not sum up to the total welfare change.

Figure A.11: Dynamic Employment Effects: Initial Job Destruction and Job Finding Rates (Kaitz Index 65%)



Notes: This figure shows the dynamic employment effect of a reform that abruptly increases the Kaitz index from 0% to 65% and the counterfactual dynamic employment effect that would obtain if, following the initial drop in employment, the average job finding rate was equal to the value in the new stationary equilibrium (red dashed line). The small difference in the initial drop relative to Figure 9 is due to the fact that this figure plots quarterly values instead of annual averages in order to show that the initial drop in employment is the same in both versions.

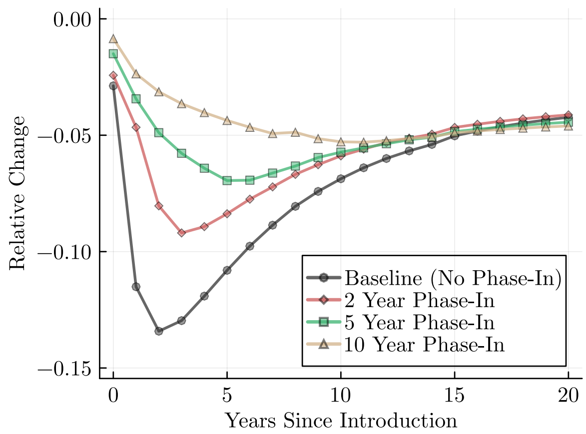
Figure A.12: Minimum Wage Path for Gradual Implementations



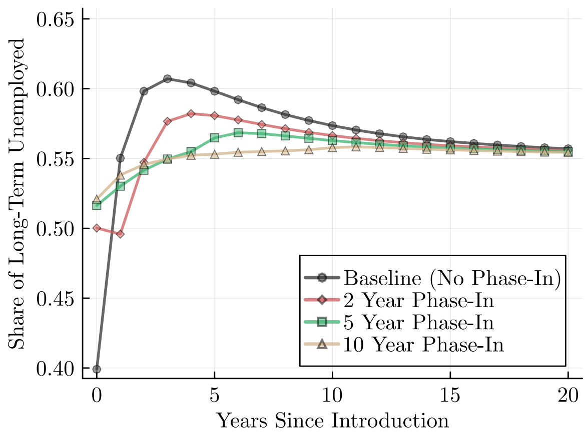
Notes: This figure shows the time path of the Kaitz index for the gradual minimum wage increases studied in Figure 11.

Figure A.13: Dynamic Effects on Job Finding Rate and Long-Term Unemployment: Gradual Implementation (Kaitz Index 65%)

(a) Job Finding Rate



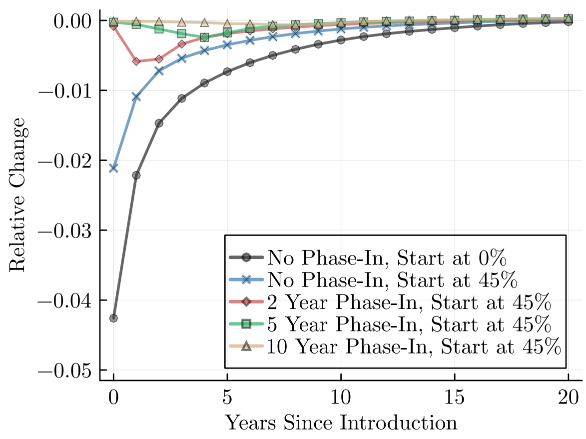
(b) Share of Long-Term Unemployed



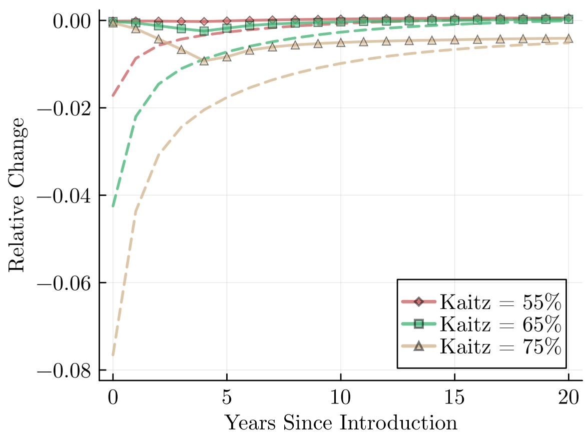
Notes: This figure shows the evolution of the change in the average job finding rate out of unemployment and the mass of long-term unemployed workers following minimum wage reforms that increases the Kaitz index from 0% to 45% on impact and then gradually to 65%.

Figure A.14: Dynamic Employment Effects: Gradual Implementation Starting from Kaitz Index of 45%

(a) Kaitz Index of 65%, Different Phase-Ins

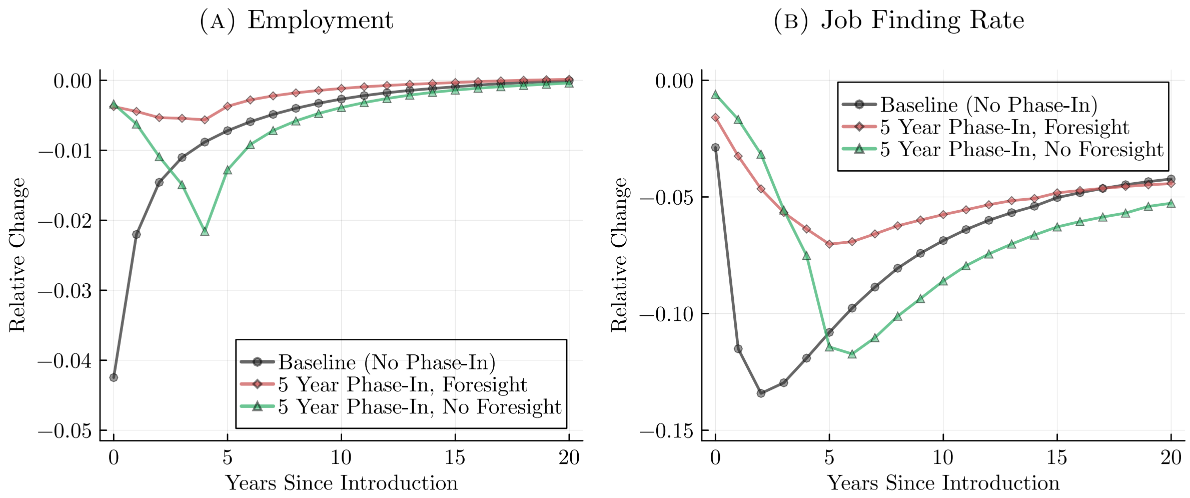


(b) Different Kaitz Indices, 5 Year Phase-In



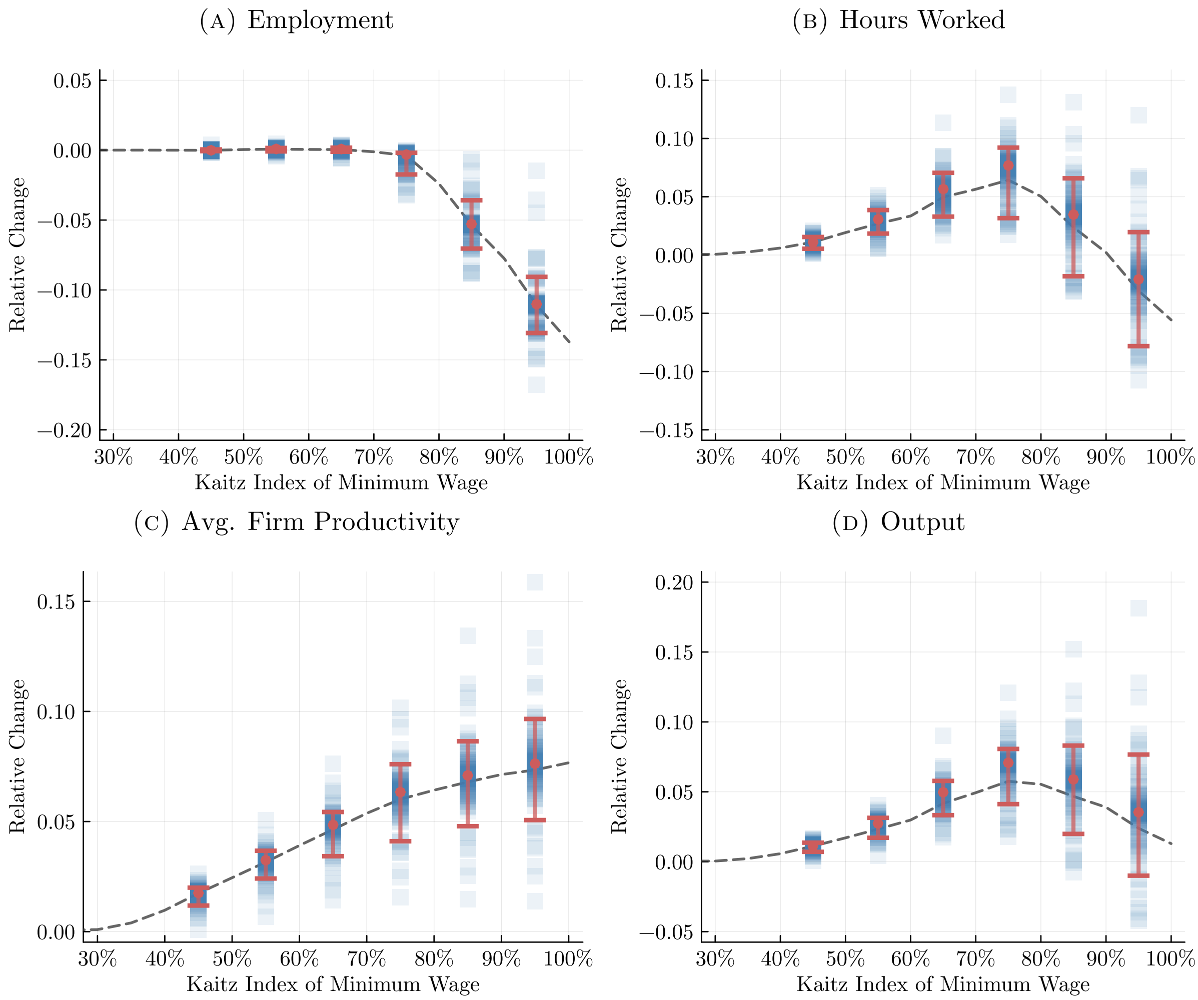
Notes: Panel A of this figure shows the evolution of the change in employment relative to the baseline employment rate in the equilibrium without a minimum wage following different minimum wage reforms that gradually increase the Kaitz index from 45% to 65% for different implementation windows. The black line shows the dynamic employment effects for an abrupt reform that increases the Kaitz index from 0% to 65% on impact (as in Panel A of Figure 9). Panel B shows the dynamic employment effects for different minimum wage reforms that gradually increase the Kaitz index from 45% to different Kaitz indices for a phase-in period of five years. The dashed lines show the effects for abrupt minimum wage hikes (from a baseline without a minimum wage).

FIGURE A.15: Dynamic Minimum Wage Effects: The Role of Expectations



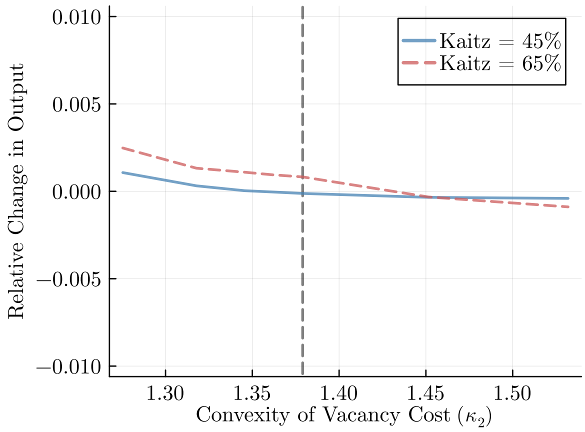
Notes: This figure shows the evolution of the change in employment and the average job finding rate following reforms that increase the Kaitz index from 0% to 65% in different ways. The black line shows the baseline transition with no phase-in period. The red and green lines correspond to gradual increases over a period of five years. The red line assumes that workers and firms have perfect foresight about the future minimum wage path (as in Panel A of Figure 11), while the green line assumes that they are surprised by each increase.

FIGURE A.16: Employment and Output Effects: Varying Internally Estimated Parameters



Notes: This figure shows the long-run effects of different minimum wages on employment, total hours worked, average firm productivity, and total output for variations of the model where one estimated parameter is increased or decreased by 5, 10, 20 or 40 percent. The hours-specific preference shifters that depend on demographics are varied all at the same time. The black dashed line shows the effects for the estimated parameters. Each blue dot represents one variation of the model, the red dot represents the median effect and the bars show the range between the 5th and 95th quantiles.

Figure A.17: Employment Effects for Different Convexities of the Vacancy Posting Cost Function (κ_2)



Notes: This figure shows the long-run employment effects for different values of κ_2 and minimum wages with Kaitz indices of 45% and 65%.

B Appendix: Model Details

Mass of Job Search The mass of search-weighted workers of type-j is denoted by S^j and the mass of all search-weighted workers in skill segment h, is

$$S(h) = \sum_j \underbrace{f(h)P(j|h)\int_\sigma \phi_\sigma \ell(\sigma|j,h)\mathrm{d}F(\sigma|j,h)}_{S^j(h)} \tag{B.1}$$

where $\ell(\cdot|j,h)$ and $F(\cdot|j,h)$ represent the optimal search effort and stationary distribution functions for type-j workers in skill segment h.

Mass of Vacancies The total mass of vacancies, $N(h)$, is given by

$$N(h) = m_f \sum_x \underbrace{\int_p v(x|p,h)\mathrm{d}\Gamma(p)}_{N_x(h)} \tag{B.2}$$

where $v(x|p,h)$ is the number of vacancies of type x posted by firms of productivity p and $N_x(h)$ is the mass of type-x vacancies.

Job Finding Probabilities For a worker in state σ, the job finding probability is the result of optimal search effort as well as the worker's acceptance decision

$$\pi^j(\ell|\sigma) = \lambda_\sigma\big(\ell\big)\mathbb{E}_{(x,p)}\Big[\mathbb{1}\{V_e^j(x,p) > V^j(\sigma)\}\Big] \tag{B.3}$$

Separation Probability A type-x employment relationship with a type-j employee ends—either due to exogenous job destruction, a Godfather shock or on-the-job search—with probability:

$$\delta^j(x,p) = \pi_{su|e_x} + \pi_{e|e_x} + \pi^j(\ell(\sigma)) \tag{B.4}$$

Hiring Probability The probability of filling a vacancy is equal to the aggregate contact rate times the probability that the contacted worker accepts the offer:

$$\eta(x,p) = \Pi(\theta)\frac{S(x,p)}{S} \tag{B.5}$$

Here, S is the total search-weighted mass of workers defined in equation B.1 and $S(x,p)$ is the mass of search-weighted workers willing to accept a type-x job at a firm with productivity p:

$$S(x,p) = \sum_j \underbrace{P_j \int_\sigma \phi_\sigma \ell(\sigma|j)\mathbb{1}\{V_e^j(x,p) > V^j(\sigma)\}\mathrm{d}F(\sigma|j)}_{\equiv S^j(x,p)} \tag{B.6}$$

Firm's Value of a Filled Vacancy. The ex-ante value of filling a vacancy of type x for a firm with productivity p is given by

$$\mathbb{E}\big[W^j(x,p)\big] = \sum_j \frac{S^j(x,p)}{S(x,p)} W^j(x,p) \tag{B.7}$$

Lifetime Utility Lifetime utility of a worker in state σ_0 is the expected value of the sum of the discounted sums of flow utilities from consumption c, search effort ℓ, and hours worked x:

$$\begin{aligned}
W(\sigma_0) &= \sum_{t=0}^{\infty} \beta^t \mathbb{E}\Big[U\Big(c(\sigma_t), \ell(\sigma_t), x(\sigma_t)\Big)\big|\sigma_0\Big] \\
&= \sum_{t=0}^{\infty} \beta^t \mathbb{E}\Big[u(c(\sigma_t)) - d(\ell(\sigma_t)) + \nu(x(\sigma_t))\big|\sigma_0\Big] \\
&= \sum_{t=0}^{\infty} \beta^t \mathbb{E}\big[u(c(\sigma_t))|\sigma_0\big] - \sum_{t=0}^{\infty} \beta^t \mathbb{E}\big[d(\ell(\sigma_t))|\sigma_0\big] + \sum_{t=0}^{\infty} \beta^t \mathbb{E}\big[\nu(x(\sigma_t))|\sigma_0\big] \\
&= W_c + W_\ell + W_x
\end{aligned} \tag{B.8}$$

Welfare of type-(j,h) workers is defined as

$$\bar{W} = \mathbb{E}\big[W_c + W_\ell + W_x|j,h\big] = \underbrace{\mathbb{E}\big[W_c|j,h\big]}_{\bar{W}_c} + \underbrace{\mathbb{E}\big[W_\ell|j,h\big]}_{\bar{W}_\ell} + \underbrace{\mathbb{E}\big[W_x|j,h\big]}_{\bar{W}_x} \tag{B.9}$$

where the expectation is taken over the stationary distribution of endogenous labor market states for type-(j,h) workers.

Consumption Equivalent Welfare Effects In order to express the welfare effects of the minimum wage policy in terms of consumption equivalents, we compute, for each worker type (j, h), by how much theiy consumption in all labor market states would have to change in order to make the worker indifferent between the new and the old policy. The consumption equivalent welfare effect, Δ, of the minimum wage policy is implicitely defined by

$$\bar{W}' = \underbrace{\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \mathbb{E}\Big[U\Big((1+\Delta)c_t, \ell_t, x_t\Big)\big|\sigma_0\Big]\right]}_{\equiv \bar{W}_0(\Delta)} \tag{B.10}$$

where all variables on the right-hand side are evaluated at baseline and $\bar{W}'$ is the worker's welfare in the new equilibrium.

In addition, let Δ_c, Δ_ℓ, and Δ_x denote the consumption equivalent welfare effect of the respective component of lifetime utility, implicitely defined by the following equations:

$$\bar{W}_c' + \bar{W}_\ell + \bar{W}_x = \bar{W}_0(\Delta_c) \tag{B.11}$$

$$\bar{W}_c + \bar{W}_\ell' + \bar{W}_x = \bar{W}_0(\Delta_\ell) \tag{B.12}$$

$$\bar{W}_c + \bar{W}_\ell + \bar{W}_x' = \bar{W}_0(\Delta_x) \tag{B.13}$$

Online Appendix

C Estimation Details

C.1 Targeted Moments

The set of moments targeted in the estimation includes the following (sets of) moments grouped into four subsets:

- overall worker moments (cum. weight: 0.1)
  - average UI-elasticity of the job finding probability of short-term unemployed workers
  - unemployment rate
  - share of long-term unemployed conditional on unemployment
  - share of part-time jobs conditional on employment
  - share of marginal employment conditional on employment
  - average job finding probability of short-term unemployed workers
  - average job finding probability of long-term unemployed workers
  - average job finding probability of full-time, part-time and marginally employed workers
- worker moments by sociodemographic group (cum. weight: 0.35)
  - unemployment rate
  - share of long-term unemployed conditional on unemployment
  - share of part-time jobs conditional on employment
  - share of mini-jobs conditional on employment
  - average job finding probability of short-term unemployed workers
  - average job finding probability of long-term unemployed workers
- firm moments: (cum. weight: 0.05)
  - mean of log firm size
  - standard deviation of log firm size
  - job vacancy rate
- distributional moments: (cum. weight: 0.5)
  - part-time and mini-job share in selected wage groups
  - male share in selected wage groups
  - $0.05, 0.1, \ldots, 0.9, 0.95$ quantiles of the wage distribution of men, women, full-time, part-time and mini-job workers
  - ratio of selected quantiles $(0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.75, 0.9, 0.95, 0.99)$ to the median of the gender specific worker fixed effect distribution and the firm fixed effect distribution of full-time, part-time and marginal workers
  - ratio of the mean firm fixed effect of part-time and marginal workers to the mean firm fixed effect of full-time workers

C.2 Data

SIAB. I mainly rely on high-quality administrative data from the IAB (Institute for Employment Research of the German Federal Employment Agency). In particular, I use the SIAB (1975-2014) data which is a two percent random sample of the integrated employment biographies collected at the IAB.[55] I use the years 2011 to 2014. The data covers all individuals in Germany, which are employed, receive unemployment benefits, are officially registered as job-seeking at the German Federal Employment Agency or (plan to) participate in programs of active labor market policies. The only workers not included in the IAB data are civil servants as they are not subject to social security contributions. Marginally employed workers, however, are included in the data even though they are also not subject to social security contributions. Information on earnings is top-coded at the social security contribution limit. This affects about 10% of all workers each year. Following Card et al. (2013), I impute top-coded earnings using Tobit regressions by year, gender, east/west, age groups and education groups.

SOEP. I complement the SIAB data with survey data from the German Socio-Economic Panel (SOEP v34, 1984-2017). The (SOEP) study is a representative longitudinal study of private households administered by the German Institute for Economic Research (DIW). The data contains annual information on more than 15 thousand workers. In the SOEP, I drop civil servants in order to be consistent with the IAB data.[56].

BHP. I also use data from the Establishment History Panel (1975-2014) provided via the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB). These data are used to compute summary statistics for the distribution of log firm size.[57]

Job Vacancy Survey. To compute the job vacancy rate (number of vacancies divided by the sum of vacancies and jobs), I use aggregate and publicly available data provided by the IAB and based on their job vacancy survey for the year 2014.[58] I multiply the number of reported vacancies by the inverse of the reporting frequency (0.5) which gives a job vacancy rate of 0.025.

C.3 Computation of Targeted Moments in the Data

Sociodemographics The distribution of sociodemographic types conditional on gender $\Pr(j|g)$ is taken from the SOEP. The distribution of gender $\Pr(g)$ is taken from the SIAB in order to use as much administrative information as possible.

[55]See Antoni et al. (2016) for a detailed description of the data. The data are confidential and can only be accessed after signing a confidentiality agreement (`https://fdz.iab.de/en/data-access/`).

[56]See Goebel et al. (2019) for a description of the data. The data are confidential and can only be accessed after signing a data distribution contract (`https://www.diw.de/en/diw_01.c.601584.en/data_access.html`).

[57]See Schmucker et al. (2016) for a description of the data. The data are confidential and can only be accessed after signing a confidentiality agreement (`https://fdz.iab.de/en/data-access/`).

[58]The data can be downloaded here: `https://www.iab.de/stellenerhebung/download` (accessed in February 2020).

Labor Market States As the SIAB data does not contain sociodemographic information for employed workers, I have to fill some gaps with information from the SOEP while ensuring that the joint distribution of gender and labor market status remains consistent with the administrative SIAB data.

I start by computing the unemployment rate conditional on j such that it is consistent with the gender-specific unemployment rate in the SIAB

$$\Pr(u|j) = \Pr(u|j, g) = \frac{\Pr(u, j|g)}{\Pr(j|g)} = \frac{\Pr(j|u, g)\Pr(u|g)}{\Pr(j|g)} \tag{C.14}$$

where only $\Pr(j|g)$ is taken from the SOEP. The probability of long-term unemployment conditional, $\Pr(lu|u, j)$, is taken directly from the SIAB.

Computing the share of type-j workers who have a type-x job requires slightly more information from the SOEP:

$$\Pr(e_x|j) = \Pr(e_x|j, g) = \frac{\Pr(j|e_x, g)\Pr(e_x|g)}{\Pr(j|g)} \tag{C.15}$$

Here, only $\Pr(j|e_x, g)$ and $\Pr(j|g)$ are taken from the SOEP.

Transition Probabilities The job finding rate out of short- and long-term unemployment, $\Pr(e'|su, j)$ and $\Pr(e'|lu, j)$, can be computed using SIAB data only. As I do not target job-to-job transition probabilities by sociodemographics, they are computed as the share of workers who change their employer or job type.

Hourly Wage Quantiles To compute hourly wages based on daily earnings reported in the SIAB data, I impute average hours worked per day using data from the SOEP and job-type dependent averages reported by Dustmann et al. (2020) who have confidential information on hours for the social security data in 2014.

The average adjusted hours for full- and part-time jobs in Dustmann et al. (2020) are almost identical to the averages in the SOEP and Structure of Earnings Survey (SES).[59] The only difference between the three data sets is that, for mini-jobs, average hours worked are substantially higher in the SOEP.

For full-time jobs, I set daily hours to 7.8 which corresponds to 39 hours per week. For par-time and mini-jobs, I use the joint distribution of hours and earnings from the SOEP to take into account that some of the variation in earnings is driven by heterogeneity in hours worked. To that end, I compute the mean and standard deviation of contractual hours worked within different earnings bins. I then draw hours worked from a Normal distribution with these parameters and impose that weekly hours for part-time and mini-jobs be in the interval $[5, 35]$

[59] Dustmann et al. (2020) adjust the raw contracted working hours in the social security records to account for differences in whether sick leave and overtime are included in the contractual hours.

and $[2, 20]$ respectively.[60] Finally I rescale the hours worked such that, on average, part-time employees work 24 hours and mini-job employees 8.7 hours per week – as reported in Dustmann et al. (2020).

Hourly wages are then computed as earnings divided by imputed hours worked. I target the 0.01, 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.7, 0.9, and 0.95 quantiles of the wage distributions conditional on job type and conditional on gender (separately). In addition, I target the share of part-time and mini-jobs and the share of men in the following five wage groups $(0, 6.5)$, $[6.5, 8.5)$, $[8.5, 12.5)$, $[12.5, 20)$, $[20, \infty)$.

Worker and Firm Fixed Effects In the absence of a minimum wage, the wage equation in my model is very simple. As in Abowd et al. (1999) (henceforth AKM), the wage w of a full-time worker employed at firm with productivity p is log-additive in her skill h and the firm's productivity

$$\log(w) = \log(r) + \log(h) + \log(p) \tag{C.16}$$

where r is the exogenous piece-rate. I estimate the empirical distribution of worker and firm-class fixed effects using a clustered AKM approach (Bonhomme et al., 2019).

While the model is consistent with an AKM-style wage equation (Abowd et al., 1999; Card et al., 2013), I do not estimate the model by straight AKM because of two distinct reasons. First, while the SIAB data is large compared to survey data sets, it covers only 2% of all workers and the firms they are employed at. This implies that the connected set of firms and workers on which firm and worker fixed effects can be identified is too small. Second, estimation would suffer from severe incidental parameter bias as the number of movers between two firms tends to be low.

Instead, I estimate the empirical distributions of worker and firm heterogeneity using the approach recently proposed by Bonhomme et al. (2019) (henceforth BLM) which solves both of these issues using dimension reduction techniques. The proposed method is particularly useful as it can be applied to data sets that cover only few firm-to-firm moves. The key assumption is that unobserved firm heterogeneity operates on the level of discrete firm classes rather than on the level of individual firms. Given an estimated partition of all firms into classes, firm class and worker fixed effects are identified from job-to-job transitions between firms of different classes rather than between different firms. This allows estimation of worker and firm (class) effects on much smaller samples of linked employer-employee data such as the SIAB (2%).

Class membership is estimated using K-means clustering that minimizes the within-class variation of within-firm earnings distributions:

$$\min_{k(1),\dots,k(J),H_1,\dots,H_K} \sum_{j=1}^{J} \frac{1}{M} \sum_{m=1}^{M} \left(F_j^m - H_{k(j)}^m\right)^2$$

[60] For part-time jobs, I use 500, 750, 1000, 1500, ..., 4000, 5000, 10000 Euro as cutoffs to define the monthly earnings bins. For mini-jobs, I use the cutoffs 100, 150, ..., 500 Euro.

where $k(j)$ is the class of firm j, F_j^m is an observable characteristic of firm j and H_k^m is the average of that characteristic across all firms in class k. I classify firms based on information on the within-firm wage distribution. In particular, I use the mean, selected percentiles (25, 50, 75) and the share of workers with top-coded earnings for full-time employees.[61] Consistent with the model where firm productivity is deterministic, I average these characteristics at the firm level over the years 2011 to 2014. This yields a time-invariant classification of firms.

Given the firm classification, I estimate the worker and firm-class fixed effects, i.e. run a clustered AKM estimation without covariates (except time fixed effects).

$$\log(w_{it}) = \alpha_i + \psi_{k(j(it))} + \gamma_t + \varepsilon_{it} \tag{C.17}$$

I then target the distribution of α conditional on gender and the worker-weighted distribution of ψ to inform the distributions of human capital and firm productivity. In particular, I target the quantile ratios $q_x^k/q_x^{0.5}$ for $k = 0.01, 0.05, 0.1, 0.3, 0.7, 0.9, 0.95, 0.99$ and $x \in \{f, p, m\}$, where q_x^k is the k-quantile of the distribution of ψ weighted by the firm's number type-x workers. In addition, I target $q_x^{0.5}/q_f^{0.5}$ for $x \in \{p, m\}$. Finally, I target the shares of the variance of log wages explained by the worker and firm components as well as the correlation between worker and firm fixed effects.

Firm Size The mean and standard deviation of log firm size are computed using administrative data from the Establishment History Panel. For consistency with the worker moments, I only consider employees between 25 and 60 years of age and drop firms that do not have employees in this age range.

Job Vacancy Rate The job vacancy rate is the number of vacancies relative to the sum of vacancies and jobs. As many vacancies are not officially registered, I do not rely on the job vacancy rate reported by Eurostat but rather use the Job Vacancy Survey (JVS).[62] The JVS contains both registered and unregistered vacancies – each account for roughly half of the total number of vacancies. In 2014, around 900 thousand vacancies were open. With roughly 36 million jobs, this gives a job vacancy rate of 2.44%.[63]

[61]This information is made available for every firm such that the within-firm earnings distribution can be approximated without observing a representative sample of employees for each firm.
[62]See Brenzel et al. (2016) for details about the data.
[63]Source: Statistics of the Federal Employment Agency.

C.4 Model Fit: Additional Tables and Figures

TABLE C.1: Model Fit: Labor Market States by Demographics

	$\Pr(e_{ft}\|e)$	$\Pr(e_{pt}\|e)$	$\Pr(e_{mj}\|e)$	$\Pr(u)$	$\Pr(lu\|u)$
Men, Single					
Data	0.832	0.100	0.068	0.109	0.526
Model	0.842	0.094	0.064	0.078	0.589
Men, Married					
Data	0.908	0.059	0.033	0.040	0.454
Model	0.902	0.068	0.029	0.052	0.506
Women, Single, No Kids					
Data	0.666	0.224	0.110	0.068	0.520
Model	0.683	0.210	0.107	0.067	0.524
Women, Single, Kids					
Data	0.330	0.534	0.136	0.140	0.552
Model	0.270	0.573	0.158	0.088	0.555
Women, Married					
Data	0.309	0.516	0.176	0.040	0.554
Model	0.276	0.559	0.165	0.076	0.536
Total					
Data	0.663	0.240	0.096	0.064	0.518
Model	0.655	0.253	0.092	0.068	0.541

Note: This table shows the share of full-time, part-time and marginal jobs conditional on employment (columns 2-4), the unemployment rate (column 5) and the share of long-term unemployment conditional on unemployment (column 6) for each sociodemographic worker type and in the population (last panel). Data: SIAB.

Table C.2: Model Fit: Job Finding Rates by Demographics

	$\Pr(e'\|su)$	$\Pr(e'\|lu)$	$\Pr(e'\|e)$
Men, Single			
Data	0.286	0.062	–
Model	0.238	0.059	–
Men, Married			
Data	0.321	0.074	–
Model	0.310	0.082	–
Women, Single, No Kids			
Data	0.321	0.065	–
Model	0.278	0.076	–
Women, Single, Kids			
Data	0.303	0.082	–
Model	0.256	0.067	–
Women, Married			
Data	0.263	0.059	–
Model	0.302	0.072	–
Total			
Data	0.296	0.067	0.035
Model	0.283	0.071	0.044

Note: This table shows the probability of finding a job out of short- and long-term unemployment as well as the job-to-job transition probability for each sociodemographic worker type and in the population (last panel). Data: SIAB.

Table C.3: Model Fit: Job-to-Job Transitions

	full-time	part-time	mini-job
Job-to-job transition			
Data	0.028	0.034	0.088
Model	0.034	0.053	0.084
Godfather shock			
Data	0.017	0.022	0.065
Model	0.017	0.022	0.064

Note: This table shows the probability of job-to-job transitions for full-time, part-time and mini-job workers. The top panel shows the probability of any job-to-job transition and the bottom panel shows the probability of being hit by the Godfather shock. Data: SIAB.

Table C.4: Model Fit: Wages

	Total		Full-Time		Part-Time		Marginal		Men		Women	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Mean	18.66	18.54	20.77	20.38	16.69	9.23	9.12	9.23	19.11	20.78	18.18	16.13
Variance (logs)	0.51	0.51	0.46	0.47	0.49	0.48	0.29	0.37	0.49	0.50	0.53	0.49
P05	7.00	6.79	8.43	8.37	6.86	6.81	5.50	4.97	7.44	7.90	6.70	6.24
P10	8.21	8.18	10.01	9.92	7.78	8.11	5.96	5.51	8.84	9.60	7.73	7.35
P15	9.32	9.32	11.47	11.21	8.65	9.13	6.41	5.92	9.95	11.00	8.67	8.25
P20	10.29	10.42	12.59	12.33	9.40	10.06	6.70	6.27	10.99	12.18	9.58	9.09
P30	12.32	12.52	14.64	14.47	10.96	11.79	7.40	6.92	13.03	14.41	11.53	10.82
P40	14.37	14.63	16.63	16.42	12.80	13.52	8.00	7.55	15.08	16.44	13.65	12.69
P50	16.66	16.67	18.89	18.36	14.81	15.30	8.69	8.20	17.06	18.56	16.01	14.70
P60	19.25	18.86	21.51	20.57	17.05	17.23	9.41	8.97	19.64	20.98	18.64	16.77
P70	22.25	21.48	24.55	23.32	19.62	19.53	10.29	9.86	22.61	23.99	21.80	19.03
P90	32.46	31.66	34.43	33.72	28.60	27.59	12.65	13.86	32.46	34.77	32.44	26.84
P95	37.60	36.00	39.43	37.27	33.65	33.28	13.95	16.86	37.60	39.19	37.60	31.88

Note: This table shows the mean wage, variance and selected percentile of hourly wages in the data and the estimated model. The variance is taken over the log wages. The moments for men and women were targeted in the estimation. Data: SIAB.

Table C.5: Model Fit: Wage Groups by Job Types and Demographics

	$[0, 6.5)$		$[6.5, 8.5)$		$[8.5, 12.5)$		$[12.5, 20)$		$[20, \infty)$	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Job Types										
Full-Time	0.011	0.007	0.039	0.048	0.146	0.156	0.345	0.367	0.459	0.422
Part-Time	0.036	0.018	0.102	0.103	0.247	0.224	0.329	0.376	0.286	0.279
Marginal	0.166	0.097	0.300	0.442	0.422	0.317	0.109	0.117	0.002	0.027
Gender										
Men	0.022	0.011	0.063	0.056	0.187	0.150	0.339	0.347	0.389	0.436
Women	0.042	0.025	0.097	0.140	0.207	0.227	0.298	0.346	0.356	0.262
Sociodemographics										
Men, Single	0.026	–	0.062	–	0.184	–	0.335	–	0.394	–
Men, Married	0.020	–	0.064	–	0.190	–	0.341	–	0.386	–
Women, Single, No Kids	0.041	–	0.087	–	0.167	–	0.316	–	0.390	–
Women, Single, Kids	0.044	–	0.106	–	0.231	–	0.270	–	0.349	–
Women, Married	0.042	–	0.101	–	0.229	–	0.292	–	0.336	–
Total	0.032	0.018	0.079	0.097	0.197	0.187	0.319	0.347	0.373	0.352

Note: This table shows the share of workeres in different wage groups conditional on job types, gender and sociodemographics. Data: SIAB.

Table C.6: Model Fit: Job Types and Demographics by Wage Groups

	$[0, 6.5)$		$[6.5, 8.5)$		$[8.5, 12.5)$		$[12.5, 20)$		$[20, \infty)$		**Total**	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
Job Types												
Full-Time	0.228	0.257	0.324	0.357	0.486	0.558	0.708	0.708	0.806	0.802	0.655	0.663
Part-Time	0.288	0.239	0.326	0.263	0.316	0.289	0.260	0.262	0.194	0.191	0.253	0.240
Marginal	0.484	0.504	0.350	0.380	0.198	0.152	0.031	0.030	0.001	0.007	0.092	0.097
Gender												
Men	0.363	0.296	0.410	0.308	0.492	0.416	0.549	0.519	0.539	0.642	0.517	0.517
Women	0.637	0.704	0.590	0.692	0.508	0.584	0.451	0.481	0.461	0.358	0.483	0.483
Sociodemographics												
Men, Single	0.171	–	0.165	–	0.197	–	0.222	–	0.224	–	0.212	–
Men, Married	0.192	–	0.245	–	0.294	–	0.326	–	0.316	–	0.305	–
Women, Single, No Kids	0.216	–	0.185	–	0.142	–	0.166	–	0.176	–	0.168	–
Women, Single, Kids	0.063	–	0.060	–	0.053	–	0.038	–	0.042	–	0.045	–
Women, Married	0.358	–	0.344	–	0.314	–	0.247	–	0.243	–	0.270	–

Note: This table shows the distribution of job-types, gender and sociodemographics within different parts of the wage distribution. Data: SIAB.

Table C.7: Model Fit: Worker Fixed Effects

	Total		**Men**		**Women**	
	Model	Data	Model	Data	Model	Data
P05 / P50	0.531	0.622	0.541	0.640	0.527	0.601
P10 / P50	0.595	0.687	0.601	0.699	0.599	0.677
P20 / P50	0.701	0.778	0.703	0.783	0.705	0.769
P30 / P50	0.797	0.854	0.798	0.854	0.800	0.850
P40 / P50	0.895	0.925	0.895	0.924	0.897	0.926
P60 / P50	1.119	1.088	1.119	1.092	1.116	1.080
P70 / P50	1.263	1.203	1.263	1.215	1.253	1.178
P80 / P50	1.450	1.370	1.453	1.393	1.429	1.314
P90 / P50	1.734	1.651	1.745	1.653	1.686	1.546
P95 / P50	1.964	1.864	1.991	1.884	1.877	1.760

Note: This table shows the median and selected percentile ratios of AKM worker fixed effects for full-time jobs. Data: SIAB.

Table C.8: Model Fit: Firm Fixed Effects

	Full-Time		Part-Time		Marginal	
	Model	Data	Model	Data	Model	Data
P50 / P50$_{ft}$	1.000	1.000	0.868	0.993	0.664	0.851
P05 / P50	0.721	0.702	0.689	0.653	0.804	0.647
P10 / P50	0.804	0.762	0.747	0.743	0.829	0.748
P25 / P50	0.922	0.877	0.869	0.857	0.894	0.867
P75 / P50	1.042	1.084	1.109	1.066	1.147	1.166
P90 / P50	1.061	1.171	1.181	1.092	1.309	1.244
P95 / P50	1.067	1.171	1.209	1.179	1.413	1.274

Note: This table shows the median and selected percentile ratios of (full-time) firm productivity for full-time, part-time and marginal jobs. The full-time firm productivity is the exponential of the AKM firm fixed effects estimated on wages of full-time workers only. Data: SIAB.

Table C.9: Model Fit: Other Moments

	Model	Data
Job Vacancy Rate		
Job Vacancy Rate	0.024	0.025
Firm Size Distribution		
Mean of log firm size	4.546	4.136
Std. dev. of log firm size	2.019	2.187

D Identification: Relationship between Model Parameters and Key Targets

This appendix shows how key moments are affected by changes in the model's parameters around their baseline values. To make the variation in parameters comparable, I multiply each parameter by $a = e^f$ for $f \in \{-0.4, -0.2, -0.1, -0.05, 0.05, 0.1, 0.2, 0.4\}$. All other parameters are kept at their baseline value. For parameters that are confined to a certain range, the adjustment factor is adjusted to keep the parameter within that range. Consider a parameter with baseline value x_0, upper bound x_{max} and lower bound x_{min}. The adjusted factor is then given by

$$\tilde{a} = \begin{cases} (x_0 + (1 - e^{-f})(x_{max} - x_0))/x_0 & \text{for } f < 0 \text{ and } x_{max} < \infty \\ (x_{min} + (e^f(x_0 - x_{min})))/x_0 & \text{for } f > 0 \text{ and } x_{min} > -\infty \\ e^f & \text{otherwise} \end{cases}$$

For the preference shifters, γ_{ft}, γ_{pt}, γ_{mj}, I do not multiply the parameters with a factor but shift them by $f \in \{-0.4, -0.2, -0.1, -0.05, 0.05, 0.1, 0.2, 0.4\}$ as some of them can be close to zero. All parameters that depend on demographics are varied simultaneously for all demographic groups.

Figure D.1: Median Full-Time Wage: Varying Key Parameters

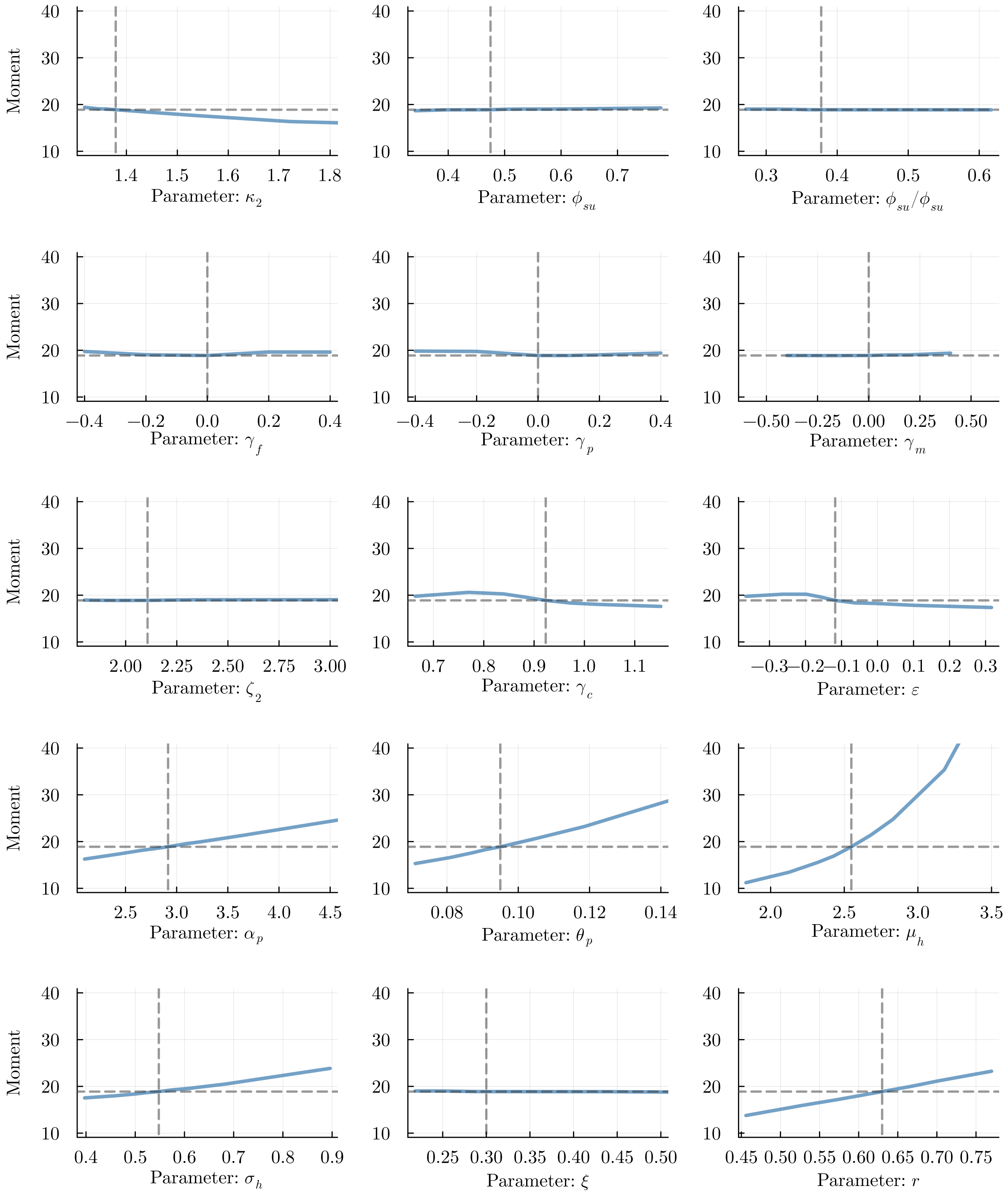


Figure D.2: Standard Deviation of Log Wages: Varying Key Parameters

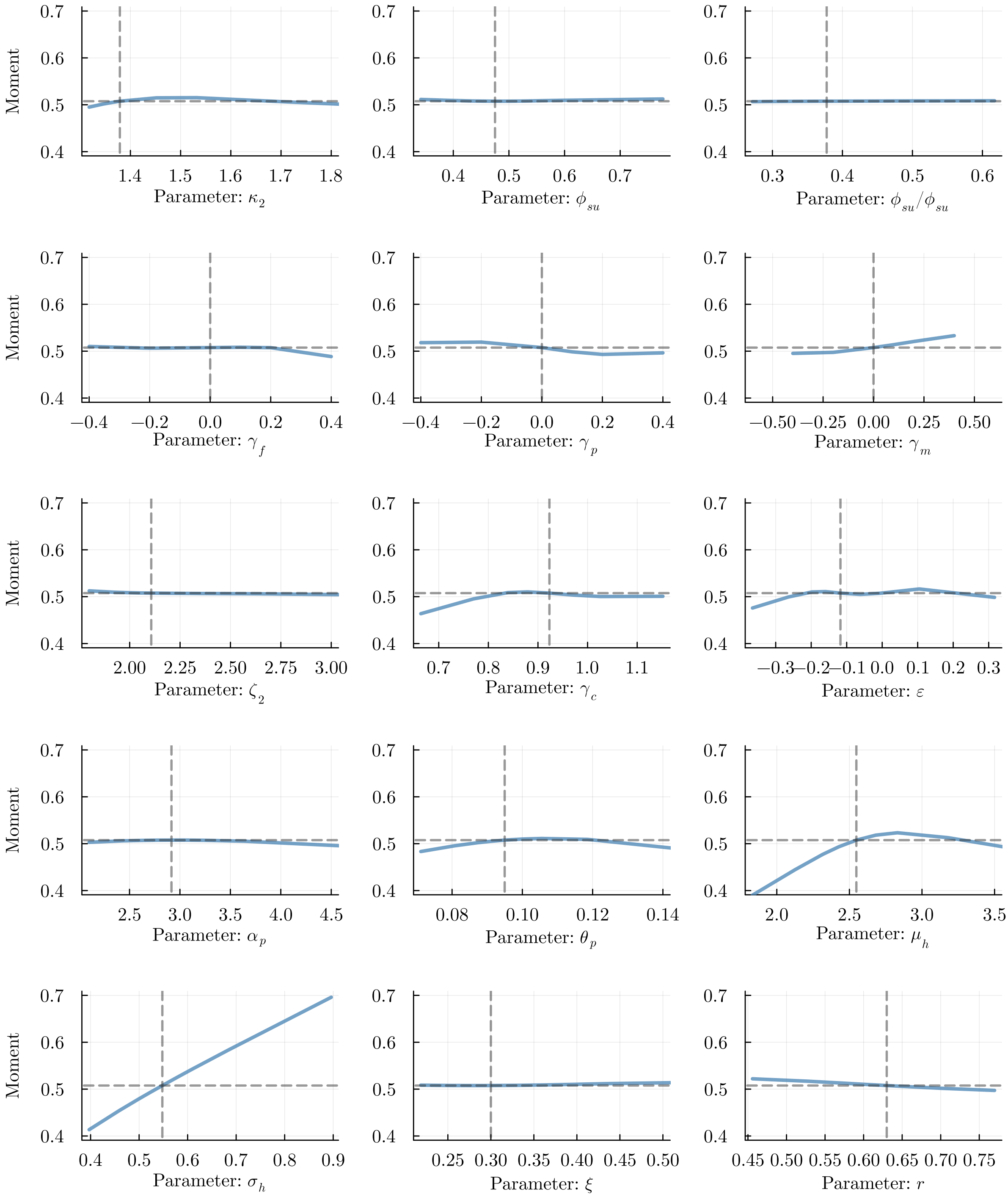


Figure D.3: Median Wage of Full-Time vs. Part-Time Jobs: Varying Key Parameters

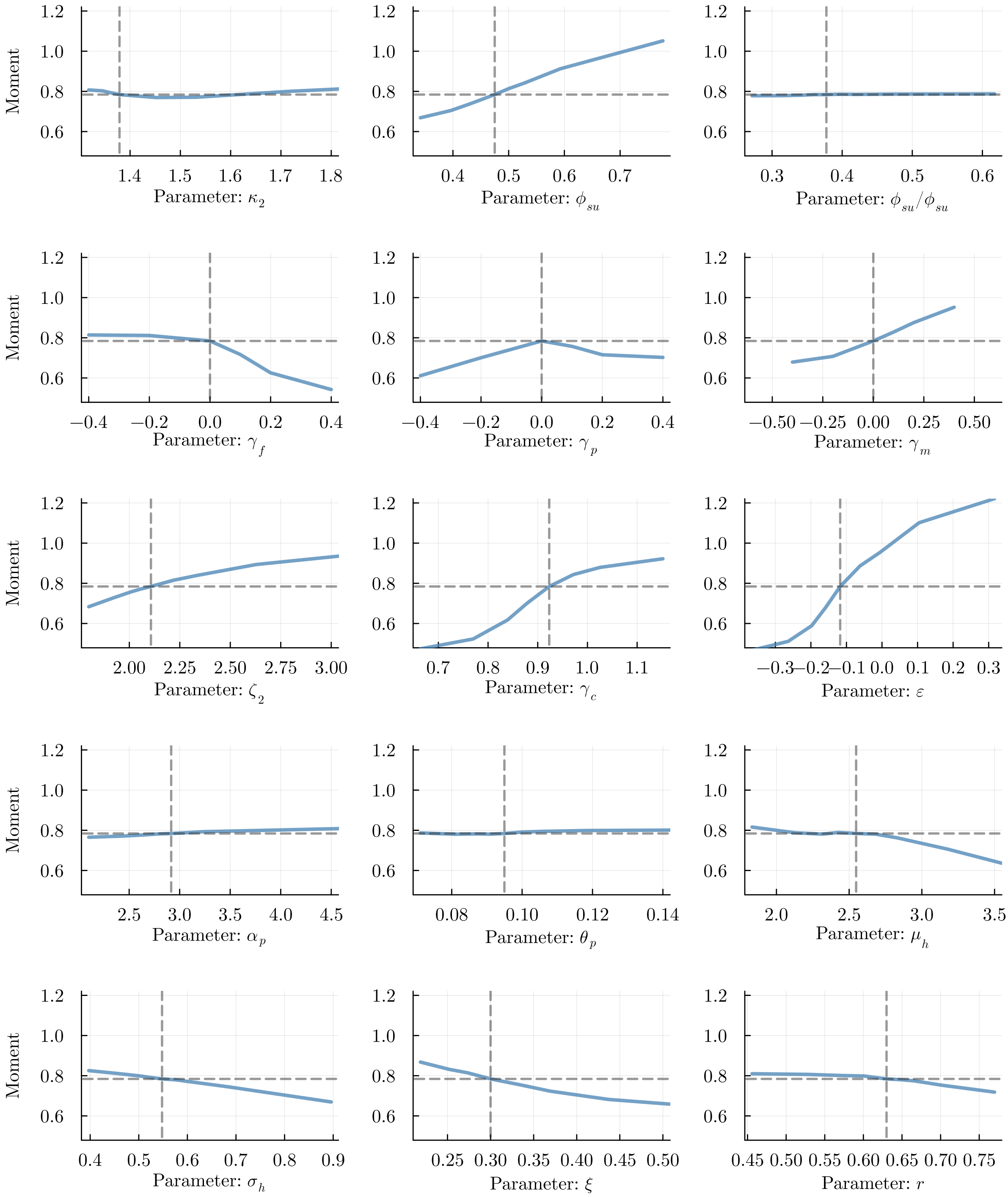


Figure D.4: Median Wage of Full-Time vs. Marginal Jobs: Varying Key Parameters

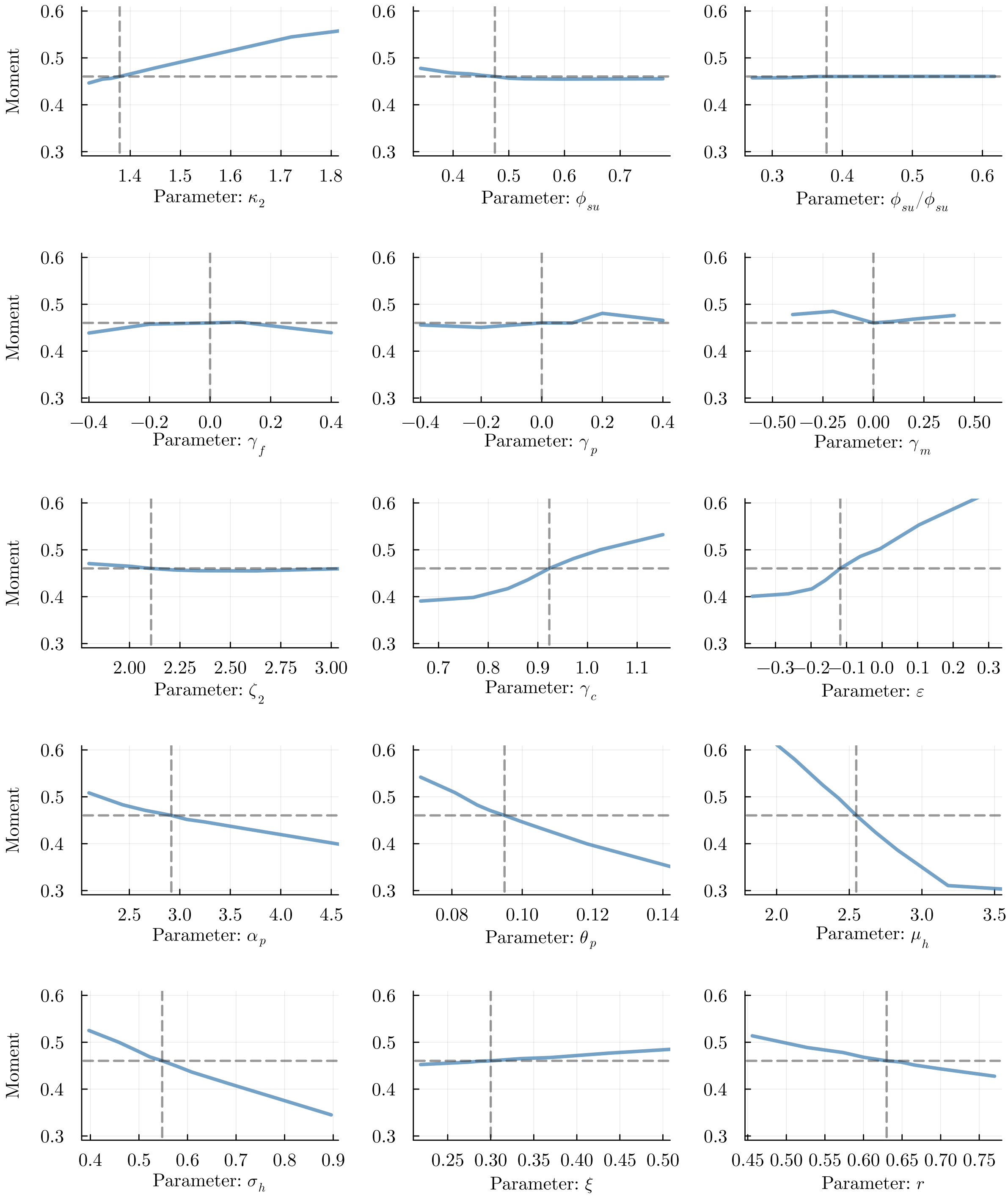


FIGURE D.5: UI-Elasticity of the Job Finding Rate: Varying Key Parameters

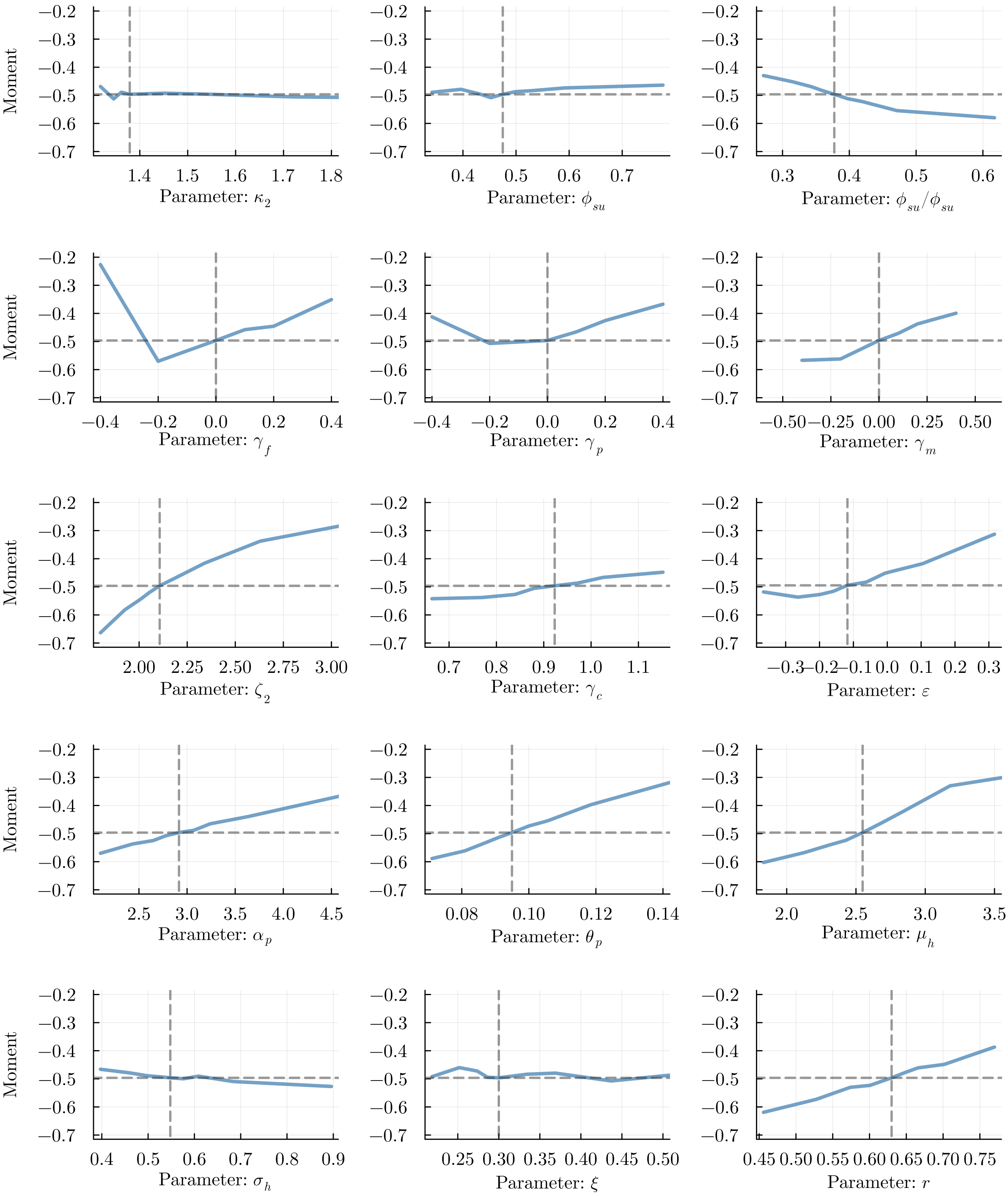


Figure D.6: Unemployment Rate: Varying Key Parameters

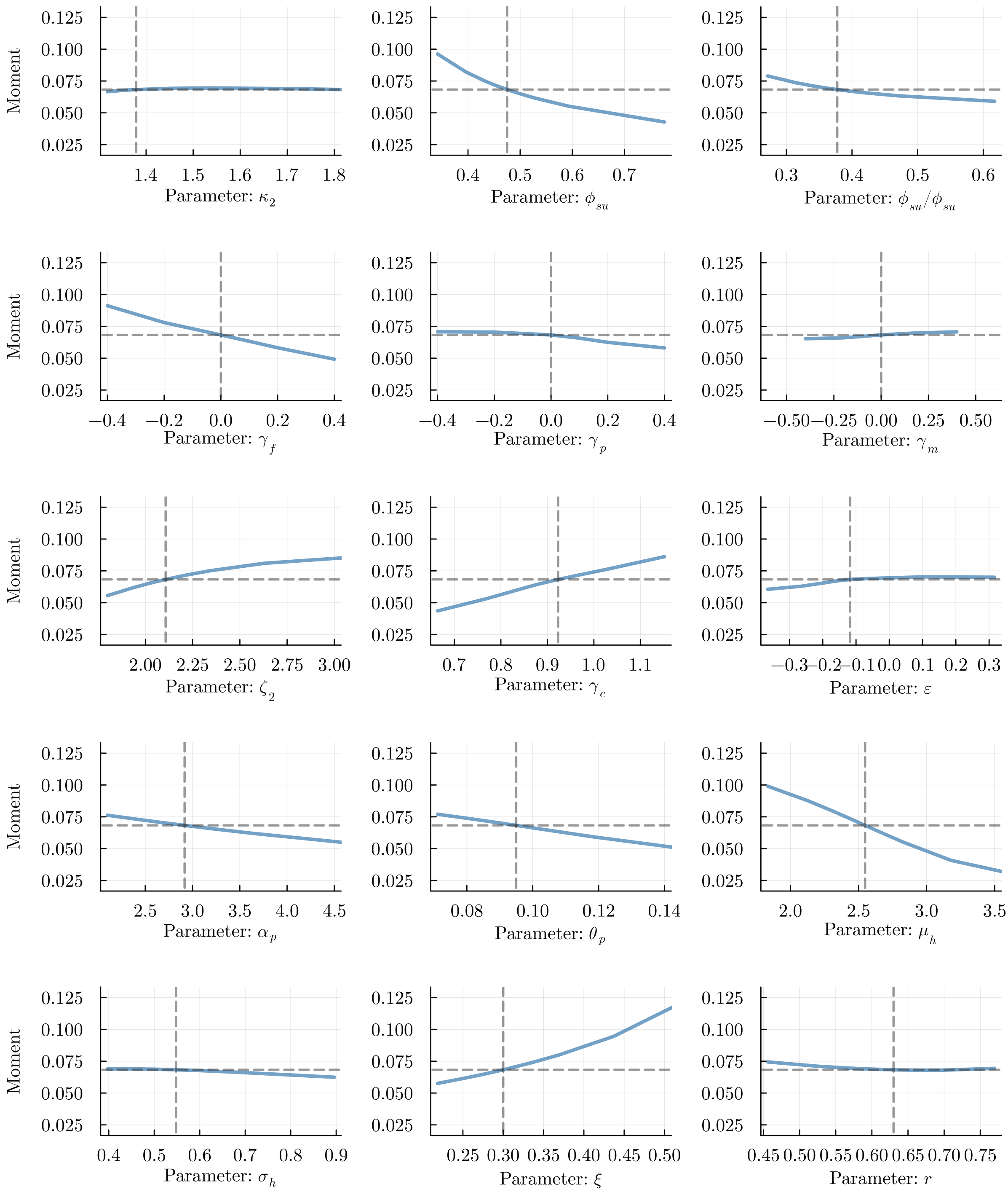


Figure D.7: Share of Part-Time Jobs: Varying Key Parameters

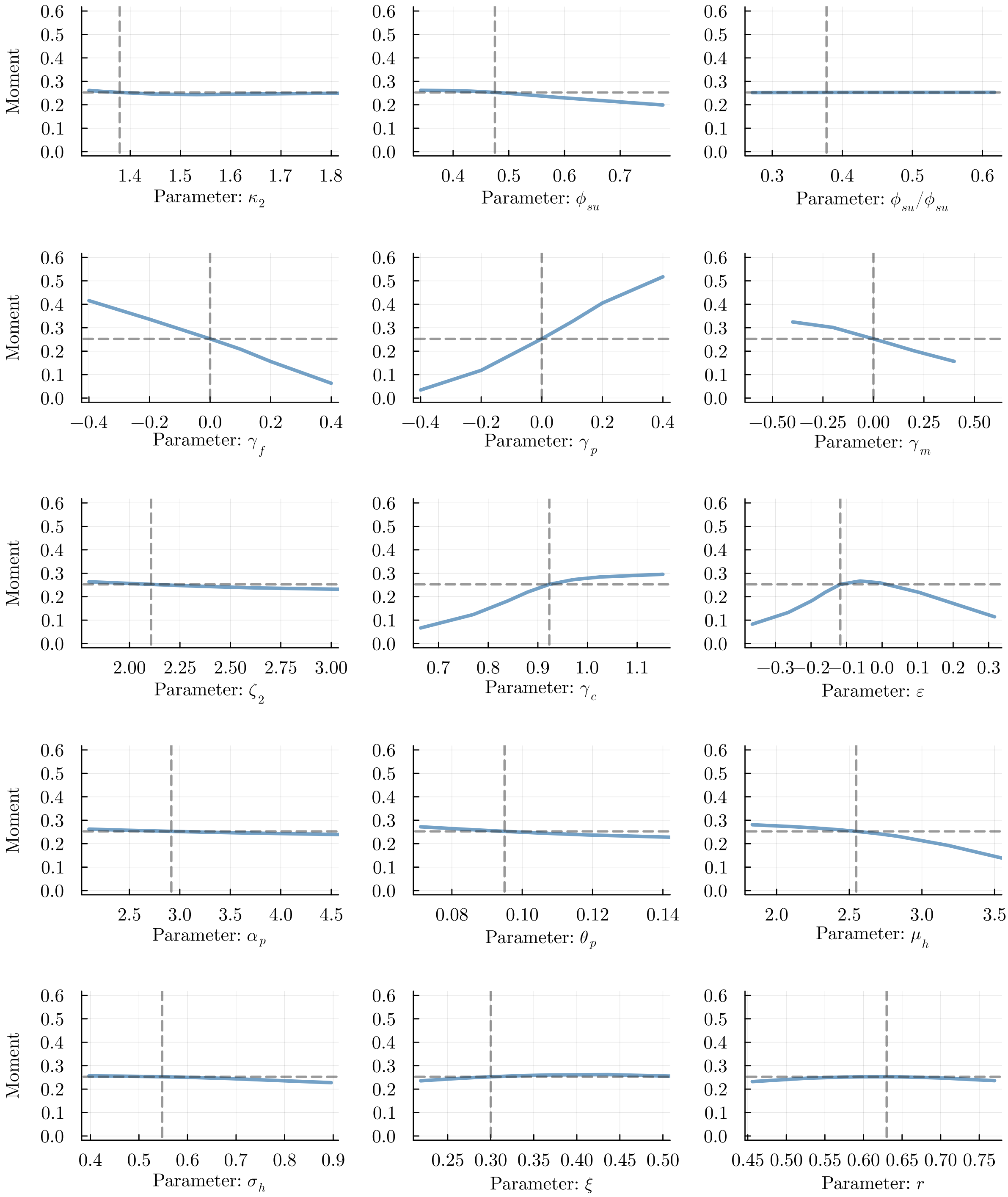


Figure D.8: Share of Marginal Jobs: Varying Key Parameters

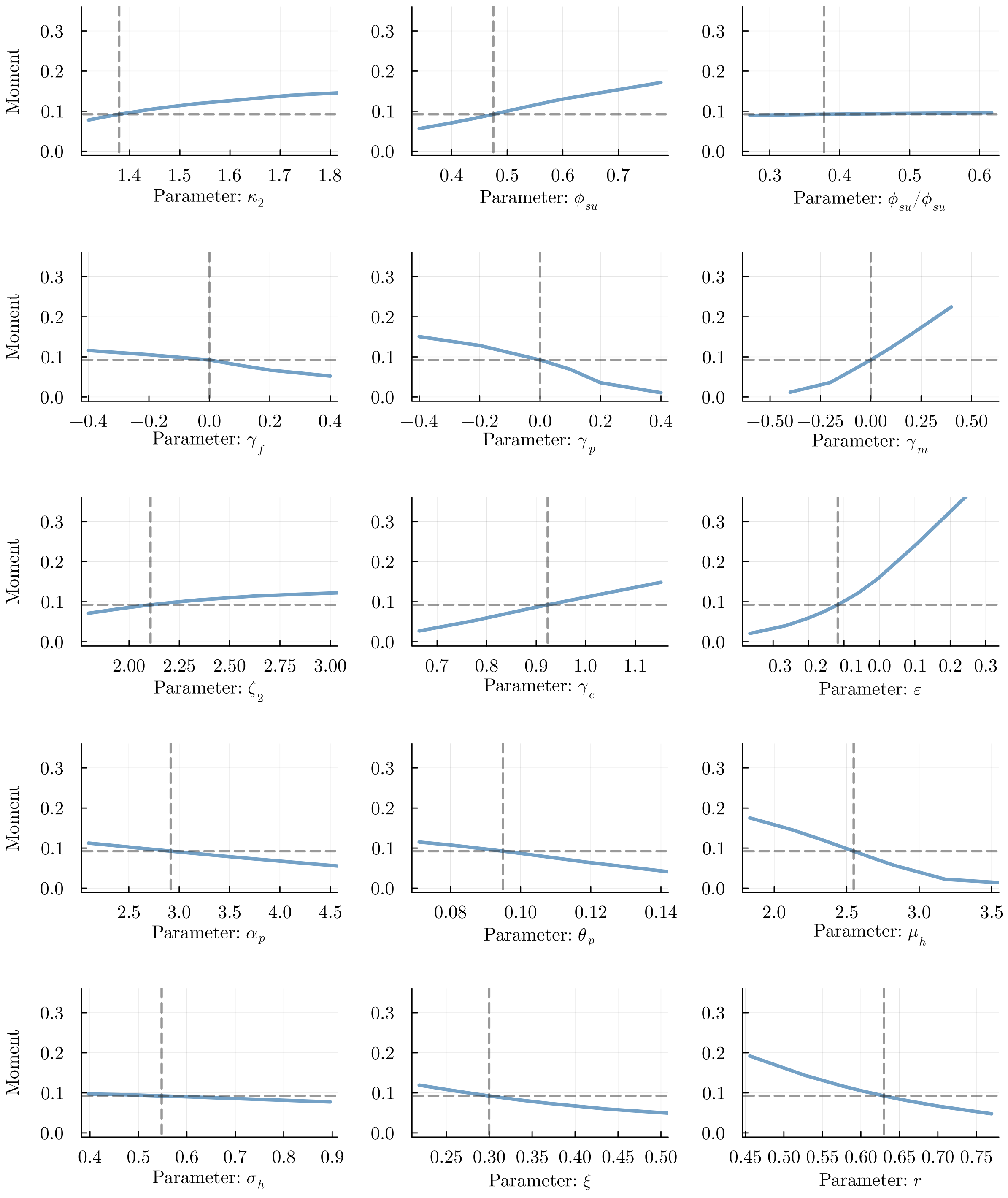


Figure D.9: Job Finding Rate out of Unemployment: Varying Key Parameters

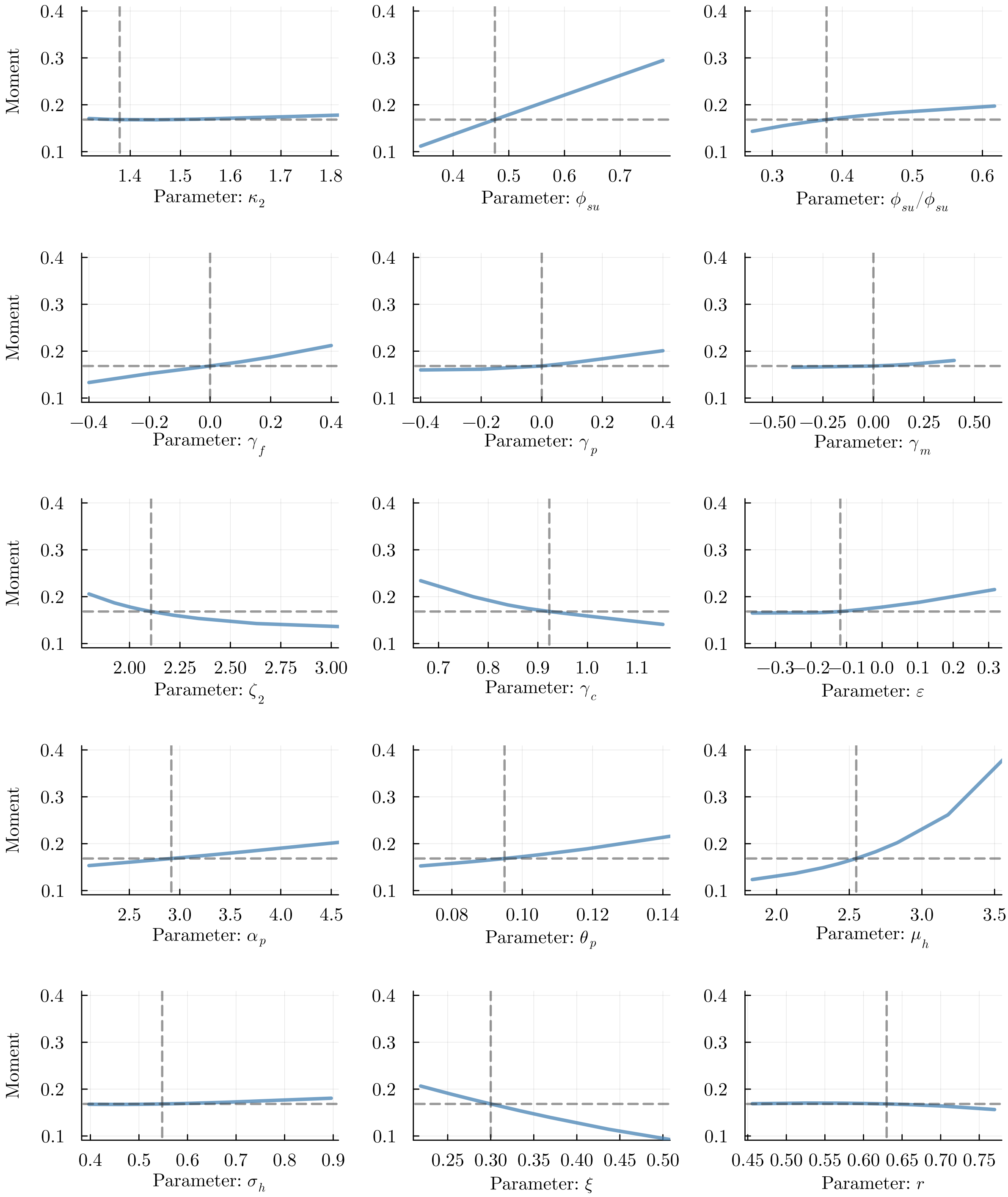


Figure D.10: Std. Dev. of Log Firm Size: Varying Key Parameters

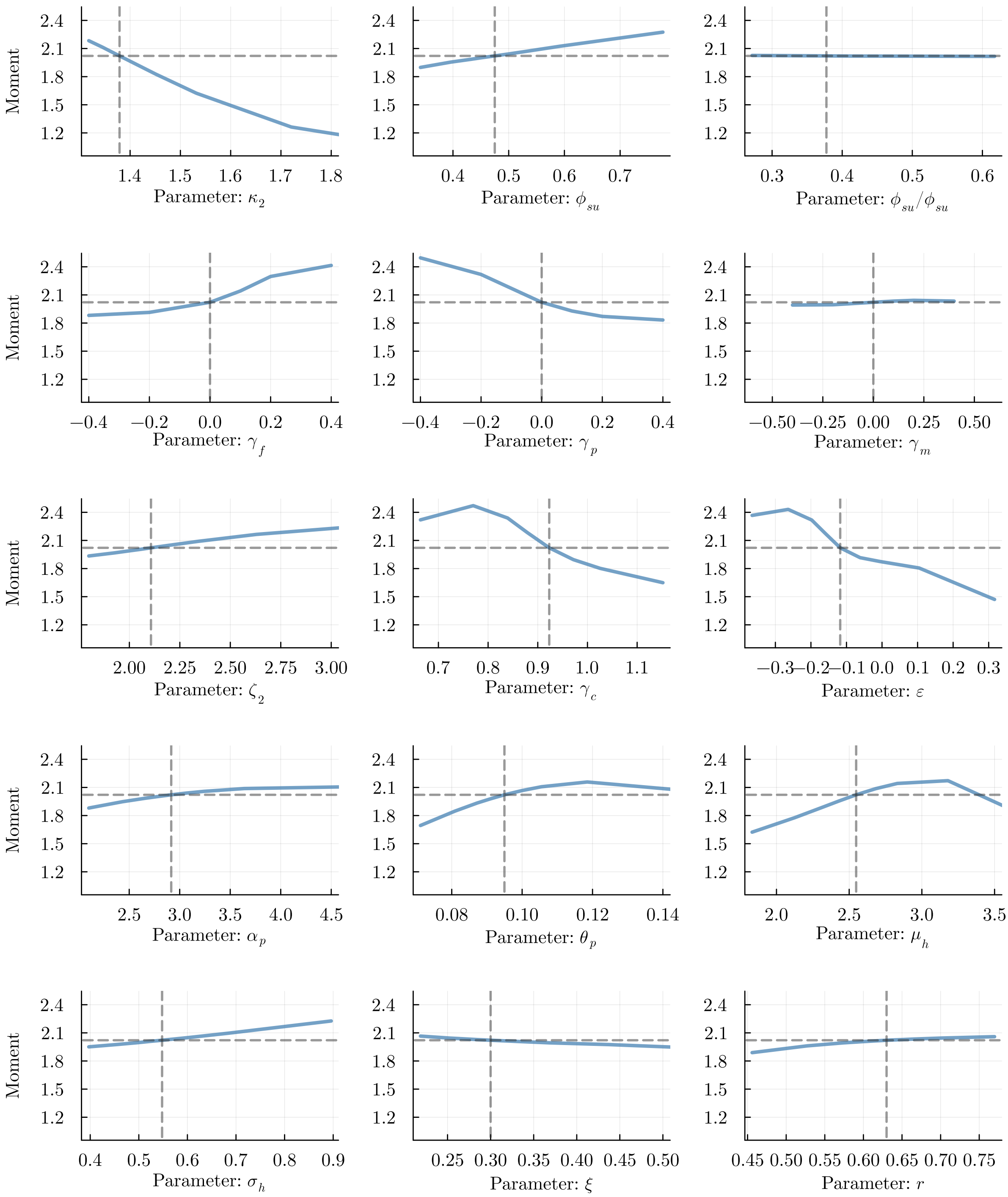
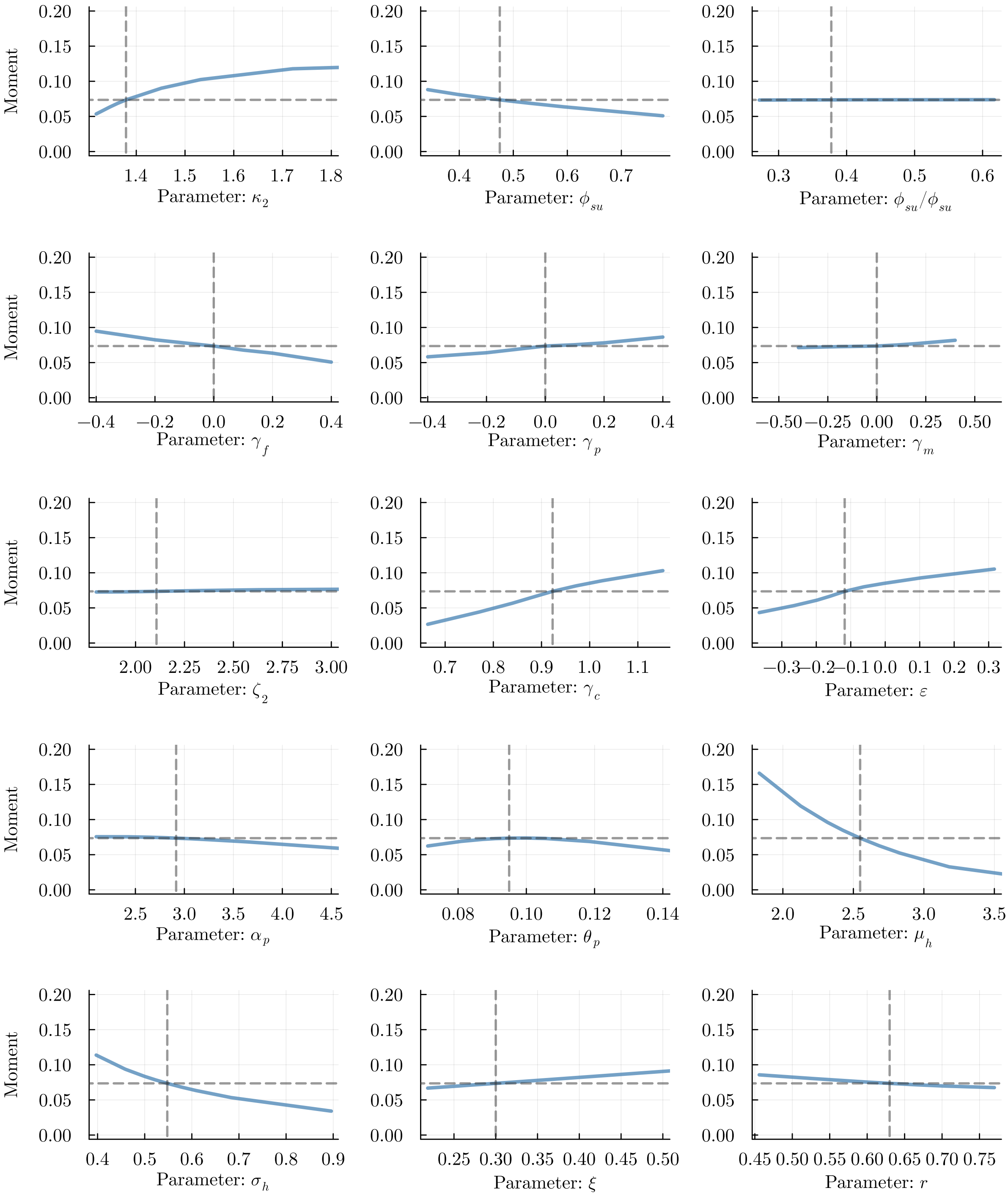


FIGURE D.11: Firm Share in Variance of Log Wages: Varying Key Parameters



E Computational Details

E.1 Steady State

In order to compute a stationary equilibrium in the economy, I discretize the state space by using a grid of values for human capital h (100 grid points) and firm productivity p (12 grid points).[64] I solve for the equilibrium in each skill-segment separately using the following algorithm:

1. Guess an initial distribution of vacancies across firm productivities and employment levels ($n^0(x,p)$), and a level of labor market tightness (θ^0).
2. Set $i = 0$
3. Taking as given the vacancy shares $n^i(x,p)$ and labor market tightness θ^i
   (a) Use equations (3), (4), (5) and (6) to solve for workers' search policies $\ell^i(j,\sigma)$ and value functions $V^i(j,\sigma)$ where σ is a point in the state space of a worker with skills h and demographics j (policy function iteration).
   (b) Compute the implied distribution of workers across states, $F(j,\sigma)$, using the search policies, equation B.3, and the exogenous transition probabilities.
   (c) Compute the implied total search mass S, the search mass willing to accept a (x,p) job offer, $S(x,p)$ and the vacancy filling probabilities $\eta(x,p)$ from equations B.1 and B.6 and the probability that a (x,p)-job filled by a type-j worker is destroyed from equation B.4.
   (d) Solve for firms' optimal vacancy policies $v^i(x,p)$ using equation (8).
   (e) Compute the implied vacancy shares $n^{i+1}(x,p)$ and labor market tightness θ^{i+1}.
4. If $n^{i+1}(x,p) \approx n^i(x,p)$ for all $x \in \{ft, pt, mj\}$ and for all p on the firm productivity grid and if $\theta^{i+1} \approx \theta^i$, stop! If not, set increment i repeat step 3!

E.2 Transition Path

Starting from the terminal stationary equilibrium, I guess a path for all equilibrium objects and solve backwards. We focus on one generic skill segment h and drop h form the notation to improve readability.

Firm Problem Assuming that the new stationary equilibrium is reached after T periods, the firm's expected value of an employment relationship with a type-j worker starting in period t is:

$$W_t^j(x,p) = y + \sum_{s=t+1}^{T-1} \beta^{s-t} \underbrace{\prod_{k=t}^{s-1} (1 - \delta_k^j(x,p))}_{\text{Pr(survival until } s)} y(x,p) + \beta^T \underbrace{\prod_{k=t}^{T-1} (1 - \delta_k^j(x,p))}_{\text{Pr(survival until } T)} W_T^j(x,p)$$

$$\text{with} \quad W_T^j(x,p) \equiv W^{j*}(x,p) = \frac{y(x,p)}{1 - \beta(1 - \delta^{j*}(x,p))}$$

[64] I verified that using more grid points for firms productivity does not change the results.

Given $W^j_{t+1}(x,p)$ and taking as given the vacancy filling rates $\eta^j_t(x,p)$, the firm optimally chooses the number of vacancies $v_t(x,p)$ to post in period t. Optimal vacancies

$$\begin{aligned}\kappa'(v_t(x,p),x,p) &= \beta^f \sum_j \eta^j_t(x,p) W^j_{t+1}(x,p) \\ &= \beta^f \Pi(\theta_t) \frac{S_t(x,p)}{S_t} \sum_j \frac{S^j_t(x,p)}{S_t(x,p)} W^j_{t+1}(x,p) \end{aligned} \tag{E.18}$$

The firm's optimal policy thus depends on the workers' search policies and distribution over labor market states via $S_t(x,p)$, S_t and $S^j_t(x,p)$. It also depends on θ_t which is a function of the other firms' policies and S_t.

Worker Problem Workers take as given next period's value functions V_{t+1} – and hence the expected surplus of finding a job – as well as the job filling rate $\Lambda(\theta_t)$ and vacancy shares $N_t(x,p)/N_t$ and choose their optimal search effort according to the resulting first order condition.

$$\frac{\mathrm{d}d^j(\ell)}{\mathrm{d}\ell} = \beta \phi_s \Lambda(\theta_t) \left(\sum_{x,p} \frac{N_t(x,p)}{N_t} \max\left\{V^j_{e,t+1}(x,p), V^j_{s,t+1}(x,p)\right\} - V^j_{s,t+1}(x,p) \right) \tag{E.19}$$

The workers' optimal policies thus depends on the firms' vacancy policies and via $N_t(x,p)$ and N_t. It also depends on θ_t which is a function of the other workers' policies and N_t.

Algorithm Focus on one skill segment h and let F_t be the distribution of workers across labor market states in period $t = 0, \dots, T$. The economy is in the initial regime until period $t = -1$. We thus set F_0 equal to the stationary distribution in the initial regime. We assume that the economy has converged to the new regime by period T. All equilibrium objects in period T are thus the equilibrium objects in the stationary equilibrium. The main backward looking object is F_t. Search mass, vacancy mass and tightness can be adjusted instantly and are thus allowed to jump from $t = 0$ to $t = 1$. The distribution F_t only jumps due to non-employability.

Knowing the initial and terminal stationary equilibrium, we proceed as follows.

1. Guess a sequence $\{F^0_t\}_t$, e.g. a piece-wise linear interpolation between F_T and F_0 taking into account the employability constraint.
2. Set $i = 0$
3. Taking as given the sequence of distributions $\{F^i_t\}_t$ as well as the value functions W^j_T and $V^j_{s,T}$, solve backwards for the equilibrium sequence of policies $\{\ell^i_t, v^i_t\}_t$. Starting with $t = T - 1$, solve for the equilibrium policies in t as follows:
   (a) guess vacancy shares and tightness: $N_t(x,p)$ and θ_t
   (b) solve for optimal search policies $\ell^i_t(j,x,p)$ using equation (E.19)
   (c) update $S_t(x,p)$, S_t, $S^j_t(x,p)$ and θ_t
   (d) solve for optimal vacancy policies $v^i_t(x,p)$ using equation (E.18)
   (e) compute implied vacancy shares and tightness

(f) if equal to guess, stop, else update guess and go back to (b)

(g) compute the workers' value: $V_t^j(\sigma) = u^j(\sigma, \ell^i) + \beta \mathbb{E}_{\sigma'|\sigma}\big[V_{t+1}^j(\sigma')|\sigma\big]$

(h) compute the firm's values: $W_t^j(x,p) = y(x,p) + \beta(1 - \delta_t^j(x,p))W_{t+1}^j(x,p)$

4. Set $t = t - 1$ and and repeat until $t = 0$
5. Use the transition matrices P_t^i to iterate forward on the distribution starting from F_0 until F_T to get $\{F_t^{i+1}\}_t$
6. Check whether the implied sequence $\{F_t^{i+1}\}_t$ differs from the guess $\{F_t^i\}_t$. Stop if yes. Set $i = i + 1$ and go back to step (3)